Magnetization plateaux of frustrated antiferromagnets

Oleg Starykh, University of Utah
Outline

• Context: new materials and promise of spin liquid

• Magnetization curve

• classical antiferromagnet in a field: entropic selection
  ⬤ spatial anisotropy - high-T stabilization of the plateau

• Quantum spins - plateau due to quantum fluctuations

• Kagome

★ Conclusions
Frustrated magnetism is growing...

Statistics from APS March 2012 Meeting

- Graphene (combined topics) 466
- Topics in topological insulators (combined topics) 206
- Pnictides (9.1.1) 163
- Frustrated magnetism (10.1.6) 145

number of abstracts
Anti-ferromagnet

\[ H = |J| \sum_{\langle ij \rangle} S_i \cdot S_j \]

Spins should be antiparallel locally
A debate

\[ H = |J| \sum_{\langle ij \rangle} S_i \cdot S_j \]

spins should be antiparallel locally

Néel

classical antiferromagnet

Landau (and Pomeranchuk)

\[ = \frac{1}{\sqrt{2}} (\uparrow \downarrow - \downarrow \uparrow) \]

singlets
Regarding the nature of excitations:

...the magnetic excitations levels correspond to the deviations from the normal distribution of the magnetic moments which are propagating through the whole crystal and are not localized in a definite place of the lattice. Such magnetic excitations will be called in the following "magnons" (this name was suggested by L. Landau).

Regarding statistics of spin excitations:

“The experimental facts available suggest that the magnons are submitted to the Fermi statistics; namely, when \( T << T_{CW} \) the susceptibility tends to a constant limit, which is of the order of \( \text{const}/T_{CW} \) [for \( T > T_{CW}, \chi = \text{const}/(T + T_{CW}) \)]. Evidently we have here to deal with the Pauli paramagnetism which can be directly obtained from the Fermi distribution. Therefore, we shall assume the Fermi statistics for the magnons.”

Ref. (5) A. Perrier and Kamerlingh Onnes, Leiden Comm. No.139 (1914)
It seems that Neel point of view has prevailed in Socorro...(and everywhere)
Singlets are back

frustrated geometry is the key

- Anderson (1973): revived the idea of singlets in the “Resonating Valence Bond” state for triangular lattice AFM

\[ \Psi = \psi_1 + \psi_2 + \ldots \]

- prototype of modern Quantum Spin Liquid
Classes of Quantum Spin Liquids

- Topological QSLs
  - full gap
- U(1) QSL
  - gapless emergent “photon”
- Algebraic QSLs
  - Relativistic CFT (power-laws)
- Spinon Fermi surface QSL
Simple models: state of affairs

- Triangular lattice AFM: ordered in a semi-classical 120° pattern, albeit with strongly reduced ordered moment (Bernu et al 1994; Capriotti et al 1999 ...)

- kagome:
  - exact diagonalization - spin liquid with very small gap to singlet excitations (Leung, Elser 1993; Lecheminant et al 1997 ...)
  - series expansions - ordered into Valence Bond Solid (no ordered moment but crystal-like order in bond variables) (Huse, Singh 2007)
  - DMRG - $Z_2$ topological spin liquid! (2011...)
Topological $\mathbb{Z}_2$ spin liquid in $s=1/2$ kagome

Spin-Liquid Ground State of the $S = 1/2$ Kagome Heisenberg Antiferromagnet

Simeng Yan, David A. Huse, Steven R. White

Identifying Topological Order by Entanglement Entropy

Hong-Chen Jiang, Zhenghan Wang, and Leon Balents

1 Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA, 93106, U.S.A.
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Nature of the Spin-Liquid Ground State of the $S = 1/2$ Heisenberg Model on the Kagome Lattice

Stefan Depenbrock, Ian P. McCulloch, and Ulrich Schollwöck
**Topological Entanglement Entropy**

- For gapped QSLs, can define a quantitative measure of long-range entanglement

\[ S(L) \sim \alpha L - \gamma \]

short-ranged spin correlations but long-range entangled state

\[ \rho_A = \text{Tr}_B |\psi\rangle \langle \psi| \]

\[ S = -\text{Tr}_A [\rho_A \ln \rho_A] \]

\[ \gamma_{\text{DMRG}} = 0.698(8) \]

\[ \gamma_{\text{th}} = \ln(2) = 0.693 \]

H. C. Jiang, Z. Wang, L. Balents
arXiv:1205.4289
Promising spin 1/2 materials

$\text{Cs}_2\text{CuBr}_4$
$\kappa-(\text{ET})_2\text{Cu}_2(\text{CN})_3$
$\text{EtMe}_3\text{Sb}[\text{Pd(dmit)}_2]_2$
$\text{Ba}_3\text{CuSb}_2\text{O}_9$

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$
$\text{Cu}_3\text{V}_2\text{O}_7(\text{OH})_2 \cdot 2\text{H}_2\text{O}$
$\text{BaCu}_3\text{V}_2\text{O}_3(\text{OH})_2$

triangular

kagome
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Magnetization studies of frustrated antiferromagnets

square lattice
AFM

no field

B

nothing interesting in magnetically ordered systems? :-(
Magnetization plateau

square

\[
\frac{\langle M \rangle}{h/J}
\]

triangular

\[
\frac{\langle M \rangle}{h/J}
\]

Classical isotropic triangular AFM in magnetic field

- No field: spiral (120 degree) state
- Magnetic field: **accidental degeneracy**
  \[ H = J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{h} \cdot \vec{S}_i \]
  \[ H = \frac{1}{2} J \sum_{\triangle} \left( \sum_{i \in \triangle} \vec{S}_i - \frac{\vec{h}}{3J} \right)^2 \]
- all states with \( \vec{S}_{i1} + \vec{S}_{i2} + \vec{S}_{i3} = \frac{\vec{h}}{3J} \) form the lowest-energy manifold
  - 6 angles, 3 equations => **2 continuous angles** (upto global U(1) rotation about \( \vec{h} \))

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Planar

Umbrella (cone)

No plateau possible
Finite $T$: minimize $F = E - T S$
Planar states have higher entropy!

H. Kawamura, S. Miyashita: 
JPSJ (1985)

Entropic Selection:
- Planar states favored by thermal fluctuations
- UUD state around $m=1/3$ resulting in quasi-plateau

Head, Griset, Alicea, OS 2010
Phase diagram of the classical model

why so stable?

Collinear UUD state does not break $U(1)$ symmetry (rotations about field axis).

Hence, no gapless Goldstone modes.

All spin excitations are gapped.

Seabra, Momoi, Sindzingre, Shannon 2011

Gvozdikova, Melchy, Zhitomirsky 2010
Experimental realizations

RbFe(MoO$_4$)$_2$: S=5/2 Fe$^{3+}$  
Smirnov et al PRB (2007)

Phase diagram contains only co-planar states (while classically - only non-planar cone state!)

Two antiferromagnetically coupled layers

Plateau width increases with T

Plateau is signaled by depression in $dM/dH$
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DMRG studies: phase diagram of 3 leg ladder

Chen, Ju, Jiang, Starykh, Balents 2012
Plateau phase

M/M_s=1/3 plateau: uud state

$R = 0.2$
M=1/3 magnetization plateau in Cs$_2$CuBr$_4$

★ Observed in Cs$_2$CuBr$_4$ (Ono 2004, Tsuji 2007) $J'/J = 0.75$

$S=1/2$

★ first observation of “up-up-down” state in spin-1/2 triangular lattice antiferromagnet

★ and 8 more phases (instead of 2 expected)!
Quantum fluctuations stabilize 1/3 plateau already at T=0
T-independent plateau width
UUD spin structure

Tsuji 2007

Shirata et al 2012
Schematic phase diagram for spin-1/2 triangular lattice

AFM

h/J

R=1-J’/J

fully polarized
C planar
IC planar
I/3
SDW
cone
quasi-collinear

many more interesting phases in addition to the plateau
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Herbertsmithite
ZnCu₃(OH)₆Cl₂

- S=1/2 kagome material does not order to 50mK with exchange J ~ 200K

Neutron scattering on single crystal Y S Lee (2011...)

S. Yan, D. Huse, S. White , 2010
Magnetization curve

classical spins: UUD plateau at 1/3 $M_{\text{sat}}$
Zhitomirsky 2002

- classical - yes
- quantum - maybe not

$s=1/2$: plateau-like feature disappears in thermodynamic limit; ramp-up behavior instead

Nakano, Sakai 2011
Experiments

plateau at $M = 0.4 \, M_{\text{sat}}$ ?!

Okamoto et al 2011
Conclusions

• This is good time for frustrated magnetism
• Spin liquids found in simple exchange spin-1/2 kagome antiferromagnet
• Magnetization plateau in spin-1/2 triangular lattice antiferromagnet: persists for all $J'/J$
• Many open theoretical questions (kagome...)
• Even more challenges for experimentalists