Panoply of orders near quantum Lifshitz point of a frustrated ferromagnet

*Oleg Starykh*
University of Utah

Life at (and near) University of Utah
Collaborators

Leon Balents, KITP, UCSB

Andrey Chubukov, FTPI, U Minnesota

Jason Alicea, Caltech
General motivation: *Exotic* but ordered phases

ordered phases

spin nematic

hidden order

quantum spin liquids

composite order parameter

\[ O^{\alpha \beta}(\mathbf{r}_i, \mathbf{r}_j) = \frac{1}{2} (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \frac{1}{3} \delta^{\alpha \beta} \langle S_i \cdot S_j \rangle \]
Quantum magnetism vs Spintronics

Spin is almost conserved

- No dipolar coupling (small magnetic moments)
  Notable exception —> spin ice

- No coupling to phonons (basically isolated system of spins)
  Notable exception —> hybridization of magnons and phonons in non-collinear spin structures

Spin transport —> mostly thermal
1) heat transport in chains/ladders
2) thermal transport in organic spin liquid candidate materials (spinon Fermi surface?)
3) magnon Hall effect (due to DM interactions)

Magnetic Coulomb Phase in the Spin Ice
Ho$_2$Ti$_2$O$_7$
Science 2009
T. Fennell$^{1,2}$, P. P. Deen$^1$, A. R. Wildes$^1$, K. Schmalzl$^3$, D. Prabhakaran$^1$, A. T. Boothroyd$^1$, R. J. Aldus$^4$, D. F. McMorrow$^4$, S. T. Bramwell$^4$

Spontaneous decays of magneto-elastic excitations in noncollinear antiferromagnet (Y$_x$Lu)$_2$MnO$_3$
Joosung Oh$^{1,2}$, Manh Duc Le$^{1,2}$, Ho-Hyun Nahm$^{1,2}$, Hasung Sim$^{1,2}$, Jaeong Jeong$^{1,2}$, T. G. Perring$^3$, Hyungie Woo$^{1,2}$, Kenji Nakajima$^1$, Seiko Ohira-Kawamura$^1$, Zohra Yamani$^1$, Y. Yoshida$^1$, H. Eisaki$^1$, S.-W. Cheong$^1$, A. L. Chernyshev$^1$, and Je-Geun Park$^{1,2}$

arxiv:1609.03262

Observation of the Magnon Hall Effect
Y. Onose$^{1,2,*}$, T. Ideue$^1$, H. Katsura$^3$, Y. Shiomi$^{1,2,4}$, N. Nagaosa$^{1,4}$, Y. Tokura$^{1,2,4}$
+ Author Affiliations
* To whom correspondence should be addressed. E-mail: onose@ap.t.u-tokyo.ac.jp
Science 16 Jul 2010;
Vol. 329, Issue 5989, pp. 297-299
DOI: 10.1126/science.1188260
Quantum magnetism vs Spintronics

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**Magnetic Coulomb Phase in the Spin Ice**

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**Observation of spin current in quantum spin liquid**

Daichi Hirobe, Masahiro Sato, Takayuki Kawamata, Yuki Shiomi, Ken-ichi Uchida, Ryo Iguchi, Yoji Koike, Sadamichi Maekawa, Eiji Saitoh

(Submitted on 21 Sep 2016)

Spin liquid is a state of electron spins in which quantum fluctuation breaks magnetic ordering while maintaining spin correlation. It has been a central topic in magnetism because of its relevance to high-T\(_c\) superconductivity and topological states. However, utilizing spin liquid has been quite difficult. Typical spin liquid states are realized in one-dimensional spin systems, called quantum spin chains. Here, we show that a spin liquid in a spin-1/2 quantum chain generates and carries spin current via its long-range spin fluctuation. This is demonstrated by observing an anisotropic negative spin Seebeck effect along the spin chains in \( \text{Sr}_2\text{CuO}_3 \). The results show that spin current can flow even in an atomic channel owing the spin liquid state, which can be used for atomic spin–current wiring.
Outline

- Magnon BEC
- Materials
  - Basic theory and some numerics
  - *Field theory of the Lifshitz point*
  - *Spin-current state* near the end-point of $1/3$ magnetization plateau
Magnon BEC and superfluidity

frustration shows up via presence of two or more degenerate minima where condensation is possible

\[ \omega \sim (k^2 - Q^2)^2 - (h_{\text{sat}} - h) \]
\[ h_{\text{sat}} = \frac{S(4J_2 - |J_1|)^2}{4J_2} \]

Condensation at one of the two minima \( \rightarrow U(1) \times Z_2 \)

\[ \langle a_k^+ \rangle = \sqrt{N} \Psi_Q \delta_{k,Q} \]
\[ \langle S_n^- \rangle \sim \Psi_Q e^{iQx_n} \]

FIG. 3. A single triangular layer of the cone state, illustrated for a field along the \( a \) axis. Circles with arrows indicate the sense of precession of the spins, as one moves along the \( x \) axis. This is most
Magnon BEC and superfluidity

frustration shows up via presence of two or more
degenerate minima where condensation is possible

1-magnon

Single magnon condensation at both minima $\rightarrow U(1)$

$E_{\text{min}}$ (supersolid)

$S^z = -1$

$\langle a_k \rangle = \sqrt{N}\psi_+ \delta_{k,Q} + \sqrt{N}\psi_- \delta_{k,-Q}$

$\langle S^-(x) \rangle = |\psi|e^{i\phi^+} \cos[Q \cdot r + \phi^-]$ $\langle S^z(x) \rangle = S - |\psi|^2 \cos^2[Q \cdot r + \phi^-]$
The difference is not small — the entire magnetization $M(h)$ of the triangular lattice antiferromagnet is determined by quantum fluctuations.

\[ \frac{E}{N} = -S \mu (\rho_1 + \rho_2) + \frac{1}{2} \Gamma_1 (\rho_1^2 + \rho_2^2) + \Gamma_2 \rho_1 \rho_2. \]
magnon superconductor

Today: condensation of magnon pairs

Formation of molecular fluid: for $d>1$ at $T=0$ this is a molecular BEC = true spin nematic (magnon superconductor)
Hidden order

No dipolar order

\[ S^z = 1 \text{ gap} \]

\[ \langle S_n^- \rangle = 0 \]

\[ \langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi} \]

Nematic order

\[ \langle S_n^- S_{n+a}^- \rangle = \Phi \neq 0 \]

\[ \langle S_n^- S_m^- \rangle = \langle S_n^x S_m^x - S_n^y S_m^y - i(S_n^x S_m^y + S_n^y S_m^x) \rangle \sim \sin^2 \theta (\cos 2\varphi - i \sin 2\varphi) \]

Magnetic quadrupole moment

think of a fluctuating fan state:

\[ \varphi \text{ is constant, while } \theta \text{ fluctuates (in time)} \]

in the interval \((\theta_0, -\theta_0)\)
Outline

- Magnon BEC
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  - Basic theory and some numerics
  - Field theory of the Lifshitz point
  - Spin-current state near the end-point of 1/3 magnetization plateau
New system: Frustrated ferromagnet

1d S=1/2 chain

\[ H = J_1 \sum_{i} S_i \cdot S_{i+1} + J_2 \sum_{i} S_i \cdot S_{i+2} - h \sum_{i} S_i^z \]

<table>
<thead>
<tr>
<th>Compound</th>
<th>( J_1 ) (K)</th>
<th>( J_2 ) (K)</th>
<th>( \angle \text{Cu-O-Cu} ) (deg)</th>
<th>( T_N ) (K)</th>
<th>( H_s ) (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Li}_2\text{ZrCuO}_4 )[12, 13]</td>
<td>-151, 35</td>
<td>94.1</td>
<td>6.4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>( \text{Rb}_2\text{Cu}_2\text{Mo}<em>3\text{O}</em>{12} )[14, 15]</td>
<td>-138, 51</td>
<td>89.9, 101.8</td>
<td>&lt; 2</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>( \text{PbCuSO}_4(\text{OH})_2 )[16–18]</td>
<td>-100, 36</td>
<td>91.2, 94.3</td>
<td>2.8</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td>( \text{LiCuSbO}_4 )[19]</td>
<td>-75, 34</td>
<td>89.8, 95.0</td>
<td>&lt; 0.1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>( \text{LiCu}_2\text{O}_2 )[20–22]</td>
<td>-69, 43</td>
<td>92.2, 92.5</td>
<td>22.3</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>( \text{LiCuVO}_4 )[23–31]</td>
<td>-19, 44</td>
<td>95.0</td>
<td>2.1</td>
<td>44.4</td>
<td></td>
</tr>
<tr>
<td>( \text{NaCuMoO}_4(\text{OH}) )</td>
<td>-51, 36</td>
<td>92.0, 103.6</td>
<td>0.59</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>

\( \beta - \text{TeVO}_4 \)

Pregelj et al., Nat.Comm.2015

K. Nawa et al, arXiv:1409.1310
multi-polar states with $9 \geq p \geq 2$
LiCuVO$_4$ : spin nematic?

\[
H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S^z_i
\]

+ weak interchain coupling $J_5$...

\[
\begin{align*}
J_1 &= -1.6 \text{ meV} \\
J_2 &= 3.9 \text{ meV} \\
J_5 &= -0.4 \text{ meV}
\end{align*}
\]
LiCuVO$_4$ experiment: collinear SDW along $\mathbf{B}$

Hagiwara, Svistov et al, 2011

Buttgen et al 2012, 2014

**FIG. 2.** Field dependence of the incommensurate wave vector $k_{ic}$ for applied magnetic fields $\mathbf{H} \parallel \mathbf{c}$ in LiCuVO$_4$. The open symbols
Evidence of a Bond Nematic Phase in LiCuVO$_4$

M. Mourigal,$^{1,2}$ M. Enderle,$^1$ B. Fåk,$^3$ R. K. Kremer,$^4$ J. M. Law,$^4,*$ A. Schneidewind,$^5$ A. Hiess,$^{1,†}$ and A. Prokofiev$^{6,7}$

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

SF = spin flip, $\Delta S = 1$
NSF = no spin flip, $\Delta S = 0$

FIG. 3 (color online). Polarized cross sections measured at $T = 70$ mK for the magnetic reflections $Q = (1, k_{1C}, 0)$ with $H \| c$ [left panels, (a)–(c)] and $Q = (0, -k_{1C}, 1)$ with $H \| a$ [right panels, (d)–(f)].
Cold reality

"Our results suggest that the theoretically predicted spin-nematic phase, if it exists in LiCuVO$_4$, can be established only within the narrow field range 40.5 < H < 41.4 T."

- so far, extensive experimental evidence for longitudinal SDW
- Spin Nematic phase is constrained to field interval < 1 T right below the saturation field (of the order 40 T)
Huge 1/3 magnetization plateau!
Phase diagram

1/3 plateau

N?

SDW

small plateau’s onset field of 27 Tesla, relative to $J \sim 100$ K, suggest the presence of ferromagnetic exchange interactions

H. Ishikawa et al, PRL 2015
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- Materials
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Frustrated ferromagnet

1d S=1/2 chain

\[ H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - h \sum_i S_i^z \]

FM

PM

\[ \frac{J_2}{(|J_1|+J_2)} \]

J₁<0 FM

J₂>0 AF

1/5

0

1
Frustrated ferromagnet

\[ J_2 \geq 0 \text{ AF} \]

\[ J_1 < 0 \text{ FM} \]

\[ H = J_1 \sum_i S_i \cdot S_{i+1} + J_2 \sum_i S_i \cdot S_{i+2} - \mu S^z_i \]

1d \( S = 1/2 \) chain

\[
\begin{align*}
\omega &\sim \frac{1}{k_x} \left( J_2 - \frac{1}{2} |J_1| \right) k_z \\
\omega &\sim \frac{1}{k_x} |J_1| k_z \\
\omega &\sim \frac{1}{k_x} \left( 4J_2 - |J_1| \right) \frac{1}{3} k_z
\end{align*}
\]

\( z = 1, 2, 4 \)

spin-wave dispersion

(1/5 - 1/2) \( J_2/(J_1+J_2) \)

Frustrated ferromagnet

\( \text{FM} \)

\( \text{PM} \)

\( 1 \)

\( 0 \)

\( M \)
Approaching from fully polarized state — Multipolar phases

\[ J_2 / (|J_1| + J_2) \]

\[ H / (|J_1| + J_2) \]

Phases with bound complexes made out of \( p \) magnons

Lifshitz point

\[ J_2 = |J_1| / 4 \]

Hikihara et al., 2008
Sudan et al., 2009

\[ \omega \sim (k^2 - Q^2)^2 - (h_{\text{sat}} - h) \]
Spin chain numerics

Frustrated ferromagnetic spin-$\frac{1}{2}$ chain in a magnetic field: 
The phase diagram and thermodynamic properties

F. Heidrich-Meisner, 1,2 A. Honecker, 3,4 and T. Vekua 5,6

We compute the ground-state energies $E_0(S_{\text{total}}^c, h=0)$ in subspaces labeled by $S_{\text{total}}^c$ on chains with periodic boundary conditions (PBC) using the Lanczos algorithm. The ground-state energies of substantially larger chains with open boundary conditions (OBC) are calculated with DMRG. Typically, we keep up to $m=400$ states in our DMRG calculations.

Then, we include the Zeeman term and obtain the field-dependent ground-state energies

$$E_0(S_{\text{total}}^c, h) = E_0(S_{\text{total}}^c, h=0) - h S_{\text{total}}^c.$$ (4)

The magnetization curves are constructed by solving the equations $E_0(S_{\text{total}}^c, h_{\text{step}}) = E_0(S_{\text{total}}^c + s, h_{\text{step}})$, which define those magnetic fields at which the magnetization increases from $M = S_{\text{total}}^c/(NS)$ to $M' = (S_{\text{total}}^c + s)/(NS)$. Steps larger than $s=1$ may occur.

s = 1/2

FIG. 1: (Color online) (a), main panel (inset): Magnetization curve $M(h)$ for $J_1 = -J_2$ ($J_1 = -2.5 J_2$). The horizontal dotted line marks $M = 1/3$. (b): $M(h)$ for $J_1 = -3 J_2$. (c): Magnetic phase diagram of the frustrated FM chain. The dotted line (with stars) marks the first-order transition between the EO phase and the $\Delta S_z = 1$ region, while the line $h = h_1$ (dashed, triangles) separates the $\Delta S_z = 1$ region from the $\Delta S_z \geq 3$ part. Uncertainties of the transition lines, e.g. due
Vector chiral and multipolar orders in the spin-$\frac{1}{2}$ frustrated ferromagnetic chain in magnetic field

Toshiya Hikihara, Lars Kecke, Tsutomu Momoi, and Akira Furusaki

\[ s = \frac{1}{2} \]

TABLE I. Number of magnons $p$ and total momentum $k$ of the multimagnon bound states which become gapless at the saturation field.

<table>
<thead>
<tr>
<th>Parameter range</th>
<th>$p$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2.669 &lt; J_1/J_2 &lt; 0$</td>
<td>2</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$-2.720 &lt; J_1/J_2 &lt; -2.669$</td>
<td>2</td>
<td>$\pi \pm \delta$ ($\delta &gt; 0$)</td>
</tr>
<tr>
<td>$-3.514 &lt; J_1/J_2 &lt; -2.720$</td>
<td>3</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$-3.764 &lt; J_1/J_2 &lt; -3.514$</td>
<td>4</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$-3.888 &lt; J_1/J_2 &lt; -3.764$</td>
<td>5</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$-3.917 &lt; J_1/J_2 &lt; -3.888$</td>
<td>6</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$-4 &lt; J_1/J_2 &lt; -3.917$</td>
<td>7</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

FIG. 9. Magnetization curves for (a) $J_1/J_2 = -2.0$, (b) $J_1/J_2 = -2.4$, (c) $J_1/J_2 = -2.5$, (d) $J_1/J_2 = -3.0$, (e) $J_1/J_2 = -3.4$, and (f) $J_1/J_2 = -3.6$. The dotted lines represent the boundaries of the regions of $\Delta S^z_{\text{tot}} = 1$ and $\Delta S^z_{\text{tot}} \geq 2$. 


Numerics: nematicity and 1st order seem connected?

1d frustrated chain

2d frustrated square lattice

1st order

2nd order

Sudan et al, 2009

Shannon, Momoi, Sindzingre PRL 2006
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Lifshitz Point

Balents, Starykh PRL 2016

- Unusual QCP: order-to-order transition
- Effective action - NLσM for unit vector $\mathbf{m}$

$$S = \int dx d\tau \left\{ i s A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\}$$

$$A_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}$$

Berry phase tune two symmetry
QCP allowed interactions at $O(q^4)$

$$\delta \propto |J_1| - 4J_2$$

All properties near Lifshitz point obey “one parameter universality” dependent upon $u/K$ ratio
Lifshitz Point

\[ S = \int dx d\tau \left\{ isA_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h\hat{m}_z \right\} \]

- Intuition: behavior near the Lifshitz point should be semi-classical, since "close" to FM state which is classical

\[
x \rightarrow \sqrt{\frac{K}{|\delta|}} x \quad \tau \rightarrow \frac{K}{\delta^2} \tau
\]

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ isA_B[\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x \hat{m}|^2 + v |\partial_x \hat{m}|^4 - \bar{h}\hat{m}_z \right\} \]

Large parameter: saddle point!

\[ v = \frac{u}{K} \quad \bar{h} = \frac{hK}{\delta^2} \]
\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ \alpha_A \hat{m} + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \hbar \hat{m}_z \right\} \]

\( v \) derives from quantum fluctuations

Need it be positive?

\[ \hat{m} \cdot \hat{m} = 1 \quad \rightarrow \quad \partial_x \hat{m} \cdot \partial_x \hat{m} = -\hat{m} \cdot \partial_x^2 \hat{m} \leq |\partial_x^2 \hat{m}| \]

Theory is stable for \( v > -1 \)

In fact, \( v < 0 \)

- Semiclassical large \( s \) limit: \( v = -\frac{3}{2s} \)
- \( s = 1/2 \) estimate: \( v_s = 1/2 = -\frac{9}{(2 + \sqrt{7})^2} \approx -0.42 \)
Saddle point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ i s A_B [\hat{m}] + \text{sgn}(\delta) |\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v |\partial_x \hat{m}|^4 - \bar{h} \hat{m}_z \right\} \]

Solution:

\[ \hat{m} = \begin{pmatrix} |\Psi| \cos(qx + \phi) \\ \pm |\Psi| \sin(qx + \phi) \\ \sqrt{1 - |\Psi|^2} \end{pmatrix} \]

Obtain

\[ q, \Psi \quad \text{versus} \quad h, v, \delta \]

many physical quantities
Saddle point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ i s A_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \right\} \]

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Obtain

\[ q, \Psi \quad \text{versus} \quad h, v, \delta \]

many physical quantities

cone state

metamagnetism
Saddle point

\[ S = \sqrt{\frac{K}{\delta}} \int dx d\tau \left\{ i s A_B[\dot{m}] + \text{sgn}(\delta) |\partial_x \dot{m}|^2 + |\partial^2 \dot{m}|^2 + v |\partial_x \dot{m}|^4 - \bar{h} \dot{m}\dot{z} \right\} \]

Note: at saddle point level there is no scale for $\delta$
**Saddle point predicts 1st order transition for $S < 6$!**

Resonances in a dilute gas of magnons and metamagnetism of isotropic frustrated ferromagnetic spin chains


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2 Physics Department and Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, D-80333 München, Germany
3 Instituto für Theoretische Physik, Georg-August-Universität Göttingen, D-37077 Göttingen, Germany
4 Instituto für Theoretische Physik, Leibniz Universität Hannover, D-30167 Hannover, Germany

(Received 22 November 2011; published 14 December 2011)

We show that spin-$S$ chains with SU(2)-symmetric, ferromagnetic nearest-neighbor and frustrating antiferromagnetic next-nearest-neighbor exchange interactions exhibit metamagnetic behavior under the influence of an external magnetic field for small $S$, in the form of a first-order transition to the fully polarized state. The corresponding magnetization jump increases gradually starting from an $S$-dependent critical value of exchange couplings and takes a maximum in the vicinity of a ferromagnetic Lifshitz point. The metamagnetism results from resonances in the dilute magnon gas caused by an interplay between quantum fluctuations and frustration.

$$v = -\frac{3}{2S} \quad \Rightarrow \quad \text{find } S_{cr} = 6$$

Thus, $S_{cr} = 5$ is the critical value of spin where the metamagnetic behavior vanishes in isotropic chains (for $S > 5$, it exists only in the presence of an easy-axis anisotropy). We would like to note that Ref. 22 reported a slightly different value of $S_{cr} = 6$; this discrepancy is due to the fact that Ref. 22 used just the leading term in the large-$S$ expansion, while our present approach is exact to all orders in $1/S$. Figure 6 illustrates the behavior of $S_{cr}$ as a function of the order of the $1/S$ expansion.
J_1 - J_2

saddle point

misses metamagnetic endpoint
and multipolar phases

\[ \mathcal{E}_{FM} = \mathcal{E}_{cone} \]

\[ \epsilon_1 = 0 \]
Metamagnetic endpoint?

Quantum corrections penalize $E_{\text{cone}}$ but not $E_{\text{FM}}$

$$\frac{\hbar}{K}$$

$$\mathcal{E}_{\text{FM}} = \mathcal{E}_{\text{cone}}$$

$$\epsilon_1 = 0$$

$$\mathcal{E}_{\text{FM}} - \mathcal{E}_{\text{cone}} \sim a\delta^2$$

$$\sim \mathcal{E} \times \sqrt{\delta/K}$$

$$\Delta \mathcal{E}_{\text{cone}} = +f(v)\delta^{5/2}$$
Quantum corrections

\[ S = \int dx d\tau \left\{ i s A_B[\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\} \]

transformation to rotating frame \[ \hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}_{\text{saddle-point}} \]

\[ \hat{m} = \sqrt{2 - \frac{\bar{\eta}\eta}{s}} \left[ \frac{\bar{\eta} + \eta}{2\sqrt{s}} \hat{e}_1 + i \frac{\bar{\eta} - \eta}{2\sqrt{s}} \hat{e}_2 \right] + (1 - \frac{\bar{\eta}\eta}{s}) \hat{e}_3, \]

effective Bogoliubov Hamiltonian

\[ S = S_{\text{sp}} + \int dx d\tau \left\{ \bar{\eta} \partial_\tau \eta + H(\bar{\eta}, \eta) \right\} + O(\eta^3) \]
diagonalization gives correction to GS energy
Metamagnetic endpoint?

$\frac{h}{K}$

$E_{FM} = E_{cone}$

$\epsilon_1 = 0$

$E_{FM} - E_{cone} \sim a\delta^2 - f(v)\delta^{5/2}$

Corrected first order curve bends slightly downward to intersect second order line
Instabilities

- Choose $E_{FM}=0$

What about multi-particle instabilities?
Instabilities

- Choose $E_{FM}=0$

Numerics suggests multipolar condensates beyond $\delta_c$
2-magnon check of the proposed scenario

- Compute exact 2-magnon energy in QFT

\[ H = \sum_k \epsilon_k \bar{\eta}_k \eta_k + \frac{1}{2L} \sum_{k_{pp'}} V(k_{/2+p},p_{/2-p}) \bar{\eta}_{k_{/2+p}} \eta_{k_{/2-p}} \eta_{k_{/2-p}} \eta_{k_{/2+p}} \]

\[ \epsilon_k = (h + 2\kappa k^4 - 2\delta k^2)/s \]

\[ V(k,p,q) = \frac{1}{s^2} \left[ \frac{1}{2} \delta k^2 - \frac{1}{8} \kappa (1 + 4v) k^4 - \delta (p^2 + q^2) + \kappa (p^4 + q^4 + \frac{1}{2} (-3 + 4v) k^2 (p^2 + q^2) + 4(3 - 2v) p^2 q^2) \right] \]

Separation of metamagnetism and multipole formation
Summary

Lifshitz point is a “parent” of many phases

\[
S = \int dx d\tau \left\{ i s A_B [\hat{m}] + \delta |\partial_x \hat{m}|^2 + K |\partial_x^2 \hat{m}|^2 + u |\partial_x \hat{m}|^4 - h \hat{m}_z \right\}
\]
\[ \mathbf{d} > 1 \]

\[
S = \int dxd^{d-1}yd\tau \left\{ isA_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + c|\partial_y \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \right\}
\]

- **Rescaling:**

\[
x \to \sqrt{\frac{K}{|\delta|}}x \quad \tau \to \frac{K}{\delta^2} \tau \quad y \to \frac{\sqrt{cK}}{\delta} y
\]

\[
S = \frac{\sqrt{K^dC^{d-1}}}{\delta^{d-1/2}} \int dxd^{d-1}yd\tau \left\{ isA_B[\hat{m}] + \text{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + |\partial_y \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \right\}
\]

**: Similar theory applies in \( \mathbf{d} > 1 \), and very similar conclusions apply**
Outline

- Magnon BEC
- Materials
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of $1/3$ magnetization plateau
Question

- Is magnon pairing possible in a system with purely repulsive (antiferromagnetic) interactions?

Nematic — superconductor analogy suggests positive answer: Magnon analogue of Kohn-Luttinger mechanism (e.g. pairing due to repulsive interactions)
**THIS TALK:**

2-magnon condensate near the end-point of the 1/3 magnetization plateau

\[ H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]

---

OAS, Reports on Progress in Physics 78, 052502 (2015),
Spatially anisotropic model: classical vs quantum

\[ H = \sum_{ij} J_{ij} S_i \cdot S_j - h \sum_i S_i^z \]

Umbrella state: favored classically; energy gain \((J-J')^2/J\)

Planar states: favored by quantum fluctuations; energy gain \(J/S\)

The competition is controlled by dimensionless parameter

\[ \delta = S(J - J')^2 / J^2 \]
Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at $T=0$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2$$
UUD-to-cone phase transition

\[ Z_3 \to U(1) \times Z_2 \text{ or } Z_3 \to \text{smth else} \to U(1) \times Z_2? \]

\[ H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]
Low-energy excitation spectra

\[ \epsilon_{d_2} = h_{c_2} - h + \frac{9 J k^2}{4} \]
for \( \delta < 3 \)

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete \( Z_3 \). Hence, **very stable**.

\[ h_{c_2} - h_{c_1} = \frac{0.6}{2S} h_{\text{sat}} = \frac{0.6}{2S} (9JS) \]

Bose-Einstein condensation of \( d_1 \) (\( d_2 \)) mode at \( k = 0 \) leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009
Low-energy excitation spectra near the plateau’s end-point

\[ \delta = \frac{40}{3} S (1 - J' / J)^2 \] parameterizes anisotropy \( J'/J \)

Extended symmetry:
4 gapless modes at the plateau’s end-point

\[ k_0 = \sqrt{\frac{3}{10S}} \]

S >> 1

Magnetization plateau is \textbf{collinear} phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete \( Z_3 \).

Alicea, Chubukov, OS PRL 2009
Bosonization of 2d interacting magnons

\[ H_{d_1d_2}^{(4)} = \frac{3}{N} \sum_{p,q} \Phi(p,q) \left( d_{1,k_0+p}^\dagger d_{2,-k_0-p}^\dagger, -k_0 + q d_{2,k_0-q} - d_{1,k_0+p}^\dagger d_{2,-k_0-p}^\dagger, -k_0 + q d_{2,k_0-q} \right) + \text{h.c.} \]

\[ \Phi(p,q) \sim \frac{(-3J)k_0^2}{|p||q|} \]

Singular magnon interaction

Magnon pair operators

\[ \left\{ \begin{array}{ll}
\Psi_{1,p} & = d_{1,k_0+p} d_{2,-k_0-p} \\
\Psi_{2,p} & = d_{1,-k_0+p} d_{2,k_0-p} 
\end{array} \right. \]

Obey canonical Bose commutation relations in the UUD ground state

\[ [\Psi_{1,p}, \Psi_{2,q}] = \delta_{1,2} \delta_{p,q} \left( 1 + d_{1,k_0+p}^\dagger d_{1,k_0+p} + d_{2,k_0+p}^\dagger d_{2,k_0+p} \right) \rightarrow \delta_{1,2} \delta_{p,q} \]

In the UUD ground state

\[ \langle d_{1}^\dagger d_1 \rangle_{uud} = \langle d_{2}^\dagger d_2 \rangle_{uud} = 0 \]

★ Interacting magnon Hamiltonian in terms of \( d_{1,2} \) bosons =

Non-interacting Hamiltonian in terms of \( \Psi_{1,2} \) magnon pairs

Chubukov, OS PRL 2013
Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense \textit{before} single magnons $d_{1,2}$

Equations of motion for $\Psi$ - Hamiltonian

\[
\langle \Psi_{1,p}^{\dagger} - \Psi_{1,p} \rangle = \frac{6 J f_p^2}{\Omega_p} \frac{3}{N} \sum_{q} f_q^2 \langle \Psi_{2,q}^{\dagger} - \Psi_{2,q} \rangle \\
\langle \Psi_{2,p}^{\dagger} - \Psi_{2,p} \rangle = \frac{6 J f_p^2}{\Omega_p} \frac{3}{N} \sum_{q} f_q^2 \langle \Psi_{1,q}^{\dagger} - \Psi_{1,q} \rangle
\]

`Superconducting' solution with \textit{imaginary} order parameter

\[
\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\gamma}{p^2}
\]

\textbf{Instability} = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

\[
1 = \frac{1}{S} \frac{1}{N} \sum_{p} \frac{k_0}{\sqrt{|p|^2 + (1 - \delta/4)k_0^2}}
\]

\textbf{no} single particle condensate

\[
\langle d_1 \rangle = \langle d_2 \rangle = 0
\]

Chubukov, OS PRL 2013
Two-magnon condensate = Spin-current nematic state

\[ J > 0 \]
\[ J' < 0 \]

\[ \hat{h}_z \cdot \mathbf{S}_A \times \mathbf{S}_C = \hat{h}_z \cdot \mathbf{S}_C \times \mathbf{S}_B = \hat{h}_z \cdot \mathbf{S}_B \times \mathbf{S}_A \propto \gamma \]

Finite scalar (and vector) chiralities. Sign of \( \gamma \) determines sense of spin-current circulation

Spontaneously broken \( \mathbb{Z}_2 \) -- spatial inversion
[in addition to broken \( \mathbb{Z}_3 \) inherited from the UUD state]

Leads to spontaneous generation of Dzyaloshinskii-Moriya interaction

Chubukov, OS PRL 2013
Continuous transition: plateau $\rightarrow$ spin-current $\rightarrow$ cone!

\[ Z_3 \rightarrow Z_3 \times Z_2 \rightarrow U(1) \times Z_2 \]

\[ H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \delta = \frac{40}{3} S \left( \frac{J - J'}{J} \right)^2 \]
Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan, 1 Alexander O. Gogolin, 2 and Fabian H.L. Eßler 3

FIG. 3. Structure of the spin currents in the spin nematic phase.

Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar, 1 Tapan Mishra, 2 Maheswar Maji, 3 R. V. Pai, 4 Subroto Mukerjee, 3, 5 and Arun Paramekanti 2, 3, 6, 7

Spin-current phase = chiral Mott insulator

gapped single particles; but
spontaneously broken time-reversal = spontaneous circulating currents
Conclusions

Magnon pairing is a fascinating problem

Route to multipolar orders of frustrated ferromagnets
extention to d=2 problems?

Spin-current/Chiral Mott insulators
Universal emergence of the one-third plateau in the magnetization process of frustrated quantum spin chains

F. Heidrich-Meisner, I. A. Sergienko, A. E. Feiguin, and E. R. Dagotto

\[ H = \sum_i \left[ \sum_{n=1,2} J_n \left\{ \frac{1}{2} (S_i^z S_{i+n}^z + S_i^x S_{i+n}^x) + \Delta S_i^z S_{i+n}^z \right\} ight. \\
\left. - h S_i^z + D (S_i^z)^2 \right], \]

\[ s = 1, 3/2, 2 \]

FIG. 11. (Color online) Magnetization curves of frustrated spin-1 chains with an anisotropic exchange ($\Delta = 2$) for (a) $J_z = 0$, (b) $J_z = 0.2$, (c) $J_z = 0.4$, and (d) $J_z = 0.8$. DMRG results (straight lines) are for $N = 60$ sites, the dashed lines are ED results (PBC). The capital letters stand for: Néel phase N, canted phase C, double-Néel phase DN.

FIG. 12. (Color online) Magnetization curves for frustrated spin-3/2 chains with an anisotropic exchange ($\Delta = 2$) for (a) $J_z = 0$, (b) $J_z = 0.2$, (c) $J_z = 0.4$, and (d) $J_z = 0.8$. DMRG results (straight lines) are for $N = 60$ sites, the dashed lines are ED results (PBC). The capital letters stand for: Néel phase N, canted phase C, double-Néel phase DN.