Angular Momentum Problems in Disk Formation
The Standard Picture

Disks galaxies are systems in **centrifugal equilibrium**

Structure of disks is governed by **angular momentum** content

### The Three Pillars of Disk Formation

- Angular momentum originates from **cosmological torques**
- Baryons and Dark Matter acquire **identical angular momentum distributions**.
- During cooling, gas **conserves** its specific angular momentum

Gas settles in disk in centrifugal equilibrium

\[ \Sigma_{\text{disk}}(R) \iff M_{\text{bar}}(j_{\text{bar}}) \iff M_{\text{dm}}(j_{\text{dm}}) \]
The Spin Parameter

Tidal Torque Theory (second-order perturbation theory):

\[ J(t) = \int_\gamma \rho(r, t) [r(t) - r_{cm}(t)] \times [v(t) - v_{cm}(t)] \, d^3r \]

conversion to comoving variables yields:

\[ J(t) \propto a^2(t) \bar{\rho}_0 \int_\gamma [1 + \delta(x, t)] (x - \bar{x}_{cm}) \times \dot{x} \, d^3x \]

It is convenient to express the specific angular momentum, \( j_{\text{vir}} = J_{\text{vir}}/M_{\text{vir}} \), in terms of the dimensionless spin parameter

\[ \lambda \propto \frac{j_{\text{vir}}}{R_{\text{vir}} V_{\text{vir}}} \]

Numerical simulations have shown that \( \langle \lambda \rangle \simeq 0.04 \)

Using that \( j_d \propto R_d V_{\text{rot}} \), and assuming that \( V_{\text{rot}} \propto V_{\text{vir}} \), we see that

\[ R_d \propto \lambda R_{\text{vir}} \]

Thus \( \lambda^{-1} \) reflects roughly the collapse factor of the baryons.
Angular Momentum & Dark Matter

Cold Dark Matter haloes have a log–normal distribution of halo spin parameters...

Cold Dark Matter haloes have a Universal Angular Momentum distribution... 

\[ \lambda = \frac{J|E|^{1/2}}{GM^{5/2}} \propto \frac{j_{\text{tot}}}{R_{\text{vir}}V_{\text{vir}}} \]

Cosmological Torques
**Testing the Paradigm**

**TEST:** Compare angular momentum distributions of disks and CDM haloes. If standard paradigm is correct, these should be identical.

**DATA:** 14 dwarf galaxies whose rotation curves are in good agreement with CDM haloes (van den Bosch & Swaters 2001).

\[ M(<j) = 2\pi \int_0^{R_j} \Sigma_{\text{disk}}(R) R \, dR \quad \text{with} \quad j = R_j V_{\text{circ}}(R_j) \]

Disks and CDM haloes have same \( p(\lambda) \).

van den Bosch, Burkert & Swaters 2001
Angular Momentum Distributions

“Typical” Angular momentum distribution of dark matter halo.

Angular momentum distribution of disk.

Disks (of dwarf galaxies) have angular momentum distributions that are clearly different than those of cold dark matter haloes!!!
Disks that form in simulations are an order of magnitude too small

Gas looses large fraction of specific angular momentum to dark matter

Hierarchical formation & “over-cooling” are to blame

White & Navarro 1993; Navarro & Steinmetz 1999

**SOLUTIONS**

(1) Prevent Cooling: feedback, preheating  
(Weil et al. 1998; Sommer-Larsen et al. 1999)

(2) Modify Power Spectrum: WDM, BSI, RSI...  
(Sommer-Larsen & Dolgov 2001)
Disk Scaling Relations I

Observations:

• \( M_{\text{disk}} = 3.1 \times 10^9 \, h^{-2} \, M_\odot \left( \frac{V_{\text{rot}}}{100 \, \text{km s}^{-1}} \right)^{3.5} \) (Bell & de Jong 2001)

• \( j_{\text{disk}} = 3.3 \times 10^2 \, \text{km s}^{-1} h^{-1} \, \text{kpc} \left( \frac{V_{\text{rot}}}{100 \, \text{km s}^{-1}} \right)^2 \)

Theoretical Predictions:

• \( M_{\text{disk}} = f_m \left( \frac{\Omega_b}{\Omega_m} \right) M_{\text{vir}} \)

• \( j_{\text{disk}} = \sqrt{2} \, f_j \, \lambda' \, R_{\text{vir}} \, V_{\text{vir}} \)

• \( M_{\text{vir}} \propto V_{\text{vir}}^3 \)

• \( R_{\text{vir}} \propto V_{\text{vir}} \)

Example: \( \Omega_m = 0.3 \quad h = 0.7 \quad \lambda = 0.04 \quad V_{\text{rot}}/V_{\text{vir}} = 1.4 \)

\[
f_m = 0.42 \left( \frac{V_{\text{vir}}}{200 \, \text{km s}^{-1}} \right)^{1/2} \quad f_j = 0.79
\]

(see also Navarro & Steinmetz 2000)
Disk Scaling Relations II

\[ f_m = 0.42 \left( \frac{V_{\text{vir}}}{200 \text{ km s}^{-1}} \right)^{1/2} \]

\[ f_j = 0.79 \left( \frac{\lambda'}{0.04} \right)^{-1} \]

- \( M(r) \) from NFW profile with \( c = 20 \) (Navarro, Frenk & White 1997)
- \( j(r) \propto r \) from \( N \)-body simulations (Bullock et al. 2001)
Sample of \( \sim 1300 \) galaxies with H\( \alpha \) RCs (Courteau et al. 2006)

Note surface brightness independence of TF relation!
Model Description

- Exponential disk in NFW dark matter halo
- Adiabatic contraction
- Disk is split in stars and cold gas using star formation threshold density
- Bulge formation based on disk stability
- Empirical stellar mass-to-light ratios: $\Upsilon = \Upsilon(L)$

We construct samples with scatter in $c$, $\Upsilon$, $\lambda_{\text{gal}}$, and $m_{\text{gal}}$

Sampling of $M_{\text{vir}}$ is such that we reproduce observed distribution of $L$

Free parameters: $\lambda_{\text{gal}}$, $m_{\text{gal}}$, $c$ (cosmology), and $\Upsilon$ (IMF)

In addition we tune $\sigma_{\ln c}$, $\sigma_{\ln \lambda}$, $\sigma_{\ln \Upsilon}$

Finally, we can modify adiabatic contraction
Zero-Point Constraints

Dutton, vdB, Dekel & Courteau, 2006
The Model That Works

Model fits slopes, zero points and scatter of $VL$ and $RL$ relations

Model fits surface brightness independence of $VL$ relation

Model is consistent with galaxy luminosity function!!
CONCLUSIONS

Problems for the Standard Problem

★ cooling very efficient in low mass haloes at high $z$

⇒ Angular Momentum Catastrophe

★ haloes have too much low angular momentum material

⇒ Morphology Problem! Too much bulge, too little disk

★ Standard model can not fit TF zero point

⇒ Haloes are too centrally concentrated

A Revised Model for Disk Formation

(1) Disks form out of Merging Clumps

(2) Dynamical Friction and 3-Body Interactions cause Halo Expansion

(3) Disks form in subset of haloes with quiescent merger history
   This results in low $\bar{\Lambda}_{\text{gal}}$ and low $\sigma_{\ln \lambda}$

(4) Formation efficiencies of disks are low
   In MW sized halo, only $\sim 20\%$ of baryons end up in disk.