You are allowed a single 8.5” × 11” sheet of notes for the exam. Show ALL of your work for each problem. If you need more paper use the scratch paper provided and clearly label your work as pertaining to a given problem. Circle and label your final answer for each part. For quantitative problems don’t forget to write your answer with the correct number of significant figures and include units. Include diagrams when appropriate.

When you are finished, fold the exam in half along the length of the paper and write your name on the top right of the back side as well as on each page of the exam.

Problem #1 (20 points)

a) (5 pts) The acceleration of gravity due to the gravitational attraction between the earth and objects on the earth is commonly taken to be uniform with respect to height above the ground, even though it actually varies inversely with the square of the distance from the center of the earth to the object. Using the concept of significant figures, explain why we can safely assume a uniform value of \( g \) near the surface of the earth.

Answer: Elevations of a few hundred feet or so above the surface of the earth are insignificantly small compared to the radius of the earth. To three significant figures (and even more) there is no difference between \( R_E^2 \) and \((R_E + h)^2\) for small heights \(h\) above the surface of the earth.

b) (5 pts) Alice and Bob have taken up diving together. They both leave the diving platform at the same time. However, Alice runs forward so that she leaves the diving board with some initial horizontal velocity while Bob just takes a single step and falls straight down. Because the acceleration of gravity is constant both of them land in the pool at the same time. Which one of them lands in the pool with the greatest velocity, and why?

Answer: They both reach the water with the same \( y \) component of velocity, but Alice has an additional velocity component in the \( x \) direction. Therefore, Alice’s total velocity is greater.

c) (5 pts) Cannon balls exit a Elaine’s cannon at a known velocity. She is trying to aim the cannon such that it strikes a target a few hundred yards away, and she calculates two different angles for which the target will be hit (one angle is smaller than 45° and the other is larger). How does she know which angle to choose, or does it matter? Describe the motion of the cannon ball in each case.

Answer: Both will strike the target. The one launched at the angle greater than 45° will follow a high arcing trajectory, while the other will follow a more straight trajectory to the target. The high-flying cannon ball will take longer to reach the target.

d) (5 pts) Joe is in an elevator inside a very tall building and moving upward at a constant velocity. He drops a ball from shoulder-height and times it as it falls to the elevator floor. Once he reaches the top floor he gets out of the elevator and repeats the experiment. Which time is shorter, or are they the same? Explain your answer.

Answer: The time will be the same in both cases. When in the elevator, the ball starts with an initial velocity that is the same as the elevator’s velocity (relative to the building). Therefore the times are the same. Quantitatively, the two pictures are as follows.
I (inside the elevator) Let the position of the floor of the elevator be given by $y_f$ and the velocity of the elevator be $v$. The ball starts at height $h$ above the elevator floor and strikes the floor when it’s height $y_b$ is equal to $y_f$.

\[ y_f = vt \]  
\[ y_b = h + vt - \frac{1}{2}gt^2 \]  
\[ h + vt - \frac{1}{2}gt^2 = vt \]  
\[ \Rightarrow t = \sqrt{\frac{2h}{g}} \]

II (outside the elevator) The ball starts at height $h$ and reaches the floor when it’s height is 0.

\[ 0 = h - \frac{1}{2}gt^2 \]  
\[ \Rightarrow t = \sqrt{\frac{2h}{g}} \]
Problem #2 (35 points) Alice and Bob are lined up for a 80m drag race. Alice’s car can accelerate at 5.02m/s² and Bob’s car can accelerate at 3.59m/s². Alice is a little hard of hearing, so when the starting bell rings it takes her 0.521s to press on the gas (Bob leaves immediately after the bell, so he has a 0.521s head start).

a) (10 pts) How long after Alice leaves the starting line does she catch up with Bob? If you can’t solve this part of the problem and you need this value for the rest of the problem you can use Δt = 3.5s.

Answer: Let Alice’s position be given by \( x_A \) and Bob’s be \( x_B \). Alice starts late such that \( t_A = t_B - \delta t \) where \( \delta t = 0.521s \), and she catches Bob when their positions are equal.

\[
x_A = \frac{1}{2} a_A t_A^2
\]

\[
x_B = \frac{1}{2} a_B t_B^2
\]

\[
\frac{1}{2} a_A t_A^2 = \frac{1}{2} a_B (t_A + \delta t)^2
\]

\[
0 = t_A^2(a_A - a_B) - t_A(2a_B \delta t) - a_B \delta t^2
\]

\[
\Rightarrow t_A = \frac{-2a_B \delta t \pm \sqrt{4a_B^2 \delta t^2 - 4(a_A - a_B)(-a_B \delta t^2)}}{2(a_A - a_B)}
\]

\[
= \frac{2(3.59m/s^2)(0.521s) \pm \sqrt{4(3.59m/s^2)^2(0.521s)^2} - 4(5.02m/s^2 - 3.59m/s^2)(-3.59m/s^2)(0.521s)^2}{2(5.02m/s^2 - 3.59m/s^2)}
\]

\[
= \frac{3.74m/s \pm \sqrt{14.0m^2/s^2} + 4(1.43m/s^2)(0.974m)}{2.86m/s^2}
\]

\[
= 2.86s
\]

b) (10 pts) How fast is each car moving when Alice catches Bob? If you can’t solve this and need a value for the rest of the problem use \( v_A = 16.0m/s \) and \( v_B = 13.8m/s \).

Answer: We have the time it takes Alice to catch Bob. So, we just need to find out how fast each is moving at that time.

Using \( t_A \) from part (a) we have the velocities as follows.

\[
v_A = a_A t_A = (5.02m/s^2)(2.86s) = 14.4m/s
\]

\[
v_B = a_B t_B = a_B(t_A + \delta t) = (3.59m/s^2)(2.86s + 0.521s) = 12.1m/s
\]

Alternatively, if you use the time \( t = 3.5s \) you get slightly higher velocities.

\[
v_A = a_A t_A = (5.02m/s^2)(3.5s) = 18m/s
\]

\[
v_B = a_B t_B = a_B(t_A + \delta t) = (3.59m/s^2)(3.5s + 0.521s) = 14m/s
\]

c) (15 pts) How much time passes between when Alice crosses the finish line (80m from the start) and when Bob crosses the finish line?

Answer: Now we need to find the total time it takes each car to travel the 80m to the finish, keeping in mind that Alice starts \( \delta t \) after Bob.

\[
x_A = \frac{1}{2} a_A t_A^2
\]

\[
t_A = \sqrt{\frac{2x_A}{a_A}} = \sqrt{\frac{2(80m)}{5.02m/s^2}} = \sqrt{32s^2} = 5.7s
\]

\[
x_B = \frac{1}{2} a_A t_B^2
\]
\[ t_B = \sqrt{\frac{2x_B}{a_B}} = \sqrt{\frac{2(80m)}{3.59m/s^2}} = \sqrt{45} = 6.7s \]  
\[ \Rightarrow \Delta t = t_B - (t_A + \delta t) \]
\[ = 6.7s - (5.7s + 0.521s) = 0.5s \]
**Problem #3 (45 points)** The Great Wallini, a circus performer, is to be shot out of a giant cannon that is 9.85m long and set at an angle of 55° above the horizontal. He begins the stunt inside the cannon at ground level. He is to pass through a hoop 18.50m above the ground whose post is 15.50m away from the base of the cannon (that is, from Wallini’s starting point - not 15.50m from where Wallini exits the cannon). Then, he will land in a moveable safety net that is 6.90m above the ground.

a) (5 pts) Draw a picture of The Great Wallini’s motion during this stunt, labeling all variables and coordinates that you will use to solve the rest of the problem.

![Diagram of Wallini's motion](image)

b) (10 pts) What should Wallini’s speed be upon exiting the cannon in order to pass through the hoop? If you can’t solve this part of the problem and you need this value for the rest of the problem use the value $|\vec{v}_0| = 25.0m/s$.

**Answer:** We know the distance Wallini must travel in both the $x$ and $y$ directions to pass through the hoop, and we know the relationship between the $x$ and $y$ components of velocity. Therefore, we simply need to solve for $v_0$ by taking time to be the same in both the $x$ and $y$ directions.

\[ x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \]
\[ = x_0 + v_0 \cos \theta t + 0 \]
\[ \Rightarrow t = \frac{x - x_0}{v_0 \cos \theta} \]
\[ y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 \]
\[ = y_0 + v_0 \sin \theta t - \frac{1}{2}gt^2 \]
\[ y - y_0 = v_0 \sin \theta \left( \frac{x - x_0}{v_0 \cos \theta} \right) - \frac{1}{2}g \left( \frac{x - x_0}{v_0 \cos \theta} \right)^2 \]
\[ = (x - x_0) \tan \theta - \frac{g(x - x_0)^2}{2v_0^2 \cos^2 \theta} \]
\[ \Rightarrow v_0^2 = \frac{g(x - x_0)^2}{2 \cos^2 \theta ((x - x_0) \tan \theta - (y - y_0))} \]
\[ v_0 = \sqrt{\frac{(9.80m/s^2)(15.50m - 9.85m \cos 55^\circ)^2}{2 \cos^2 55^\circ((15.50m - 9.85m \cos 55^\circ) \tan 55^\circ - (18.50m - 9.85m \sin 55^\circ))}} \]
\[
\sqrt{\frac{951 \text{ m}^3/\text{s}^2}{2 \cos^2 55^\circ (14.1 \text{ m} - 10.43 \text{ m})}} = \sqrt{\frac{951 \text{ m}^3/\text{s}^2}{2.4 \text{ m}}} = 20 \text{ m/s (i.e. } 2.0 \times 10 \text{ m/s})
\]

\[c) \quad (10 \text{ pts}) \text{ What must his total acceleration be inside the cannon to ensure that he makes it? Only the magnitude of his acceleration is required, since we already know that the direction is } 55^\circ \text{ above the horizontal. If you can’t solve this part and you need this value for the rest of the problem use } |\vec{a}| = 25.0 \text{ m/s}^2.
\]

**Answer:** We know how long the cannon is, and we know the required exit velocity from the cannon.

\[
v^2 - v_0^2 = 2a\Delta x \quad (37)
\]
\[
\Rightarrow a = \frac{v^2 - v_0^2}{2\Delta x} \quad (38)
\]
\[
= \frac{(20 \text{ m/s})^2}{2(9.85 \text{ m})} = 20 \text{ m/s}^2 \quad (39)
\]

Alternatively, we can use the velocity that was given if we couldn’t solve part (b).

\[
v^2 - v_0^2 = 2a\Delta x \quad (41)
\]
\[
\Rightarrow a = \frac{v^2 - v_0^2}{2\Delta x} \quad (42)
\]
\[
= \frac{(25.0 \text{ m/s})^2}{2(9.85 \text{ m})} = 31.7 \text{ m/s}^2 \quad (43)
\]

\[d) \quad (10 \text{ pts}) \text{ Where should the safety net’s base be centered so that the Great Wallini doesn’t break his neck? If you can’t solve this part and you need this value for the rest of the problem use } x_{\text{net}} = 40 \text{ m}.
\]

**Answer:** We can use the known value for the height of the net to determine how long he is in the air, then use that time to find how far he travels in the \(x\) direction during the flight.

\[
y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \quad (45)
\]
\[
0 = (y_0 - y) + v_0 \sin \theta t - \frac{1}{2}gt^2 \quad (46)
\]
\[
\Rightarrow t = \frac{-v_0 \sin \theta \pm \sqrt{v_0^2 \sin^2 \theta - 4(-\frac{1}{2}g)(y_0 - y)}}{2(-\frac{1}{2}g)} \quad (47)
\]
\[
= \frac{(20 \text{ m/s}) \sin 55^\circ \mp \sqrt{(20 \text{ m/s})^2 \sin^2 55^\circ + 2(9.80 \text{ m/s}^2)(9.85 \text{ m} \sin 55^\circ - 6.90 \text{ m})}}{9.80 \text{ m/s}^2} \quad (48)
\]
\[
= \frac{16 \text{ m/s} \mp \sqrt{270 \text{ m}^2/\text{s}^2 + 23}}{9.80 \text{ m/s}^2} = 3.4 \text{ s} \quad (49)
\]

Again, if his initial velocity could not be determined earlier we would have a different answer (using \(v_0 = 25.0 \text{ m/s}\)).

\[
t = \frac{(25.0 \text{ m/s}) \sin 55^\circ \mp \sqrt{(25.0 \text{ m/s})^2 \sin^2 55^\circ + 2(9.80 \text{ m/s}^2)(9.85 \text{ m} \sin 55^\circ - 6.90 \text{ m})}}{9.80 \text{ m/s}^2} \quad (50)
\]
\[
= 4.23 \text{ s} \quad (51)
\]

In either case, his position in the \(x\) direction is just his \(x\) velocity times time.

\[
x = x_0 + v_{0x} t + \frac{1}{2}a_xt^2 \quad (52)
\]
\[ v = 0 + v_0 \cos \theta t + 0 \]
\[ = (20m/s)(\cos 55^\circ)(3.4s) \text{ or } (25.0m/s)(\cos 55^\circ)(4.23s) \]
\[ = 39m \text{ or } 60.8m \]  

\textbf{e) (10 pts)} What is The Great Wallini’s average velocity (a vector) for the entire trip? In other words, what is his average velocity from his starting point at the base of the cannon until he lands in the safety net? Even if you cannot solve for his average velocity you will get most of the credit for finding his displacement vector.

\textbf{Answer:} We know his starting and final positions, which gives us his displacement vector. All we need to know is the total time of the trip. To find the time we simply add the time of flight to the time spent accelerating in the cannon. Depending on which values you use for the various parts above this can give quite a range of values, so I will just use the correct numbers.

\[ \vec{v} = \frac{\Delta \vec{d}}{\Delta t} \]
\[ \Delta \vec{d} = (x - x_0)\hat{x} + (y - y_0)\hat{y} \]
\[ = 39m\hat{x} + 6.90m\hat{y} \]
\[ \Delta t = t_{\text{cannon}} + t_{\text{flight}} \]
\[ = \sqrt{\frac{2x_{\text{cannon}}}{a_{\text{cannon}}} + t_{\text{flight}}} = \sqrt{\frac{2(9.85m)}{20m/s^2} + 3.4s} = 4.4s \]
\[ \Rightarrow \vec{v} = 8.9m/s\hat{x} + 1.6m/s\hat{y} \]
Extra Credit Problem (20 points)  So far in class we have dealt with objects moving with constant acceleration. Now, let’s define the rate of change of acceleration to be the jerk \( \vec{j} = \frac{\Delta \vec{a}}{\Delta t} \). Describe in words the motion of an object moving with constant jerk and plot the velocity and acceleration of such an object vs time. If you can, give a physical example of something moving with constant jerk (either positive or negative).

Answer: An object moving with constant jerk would move with consistently increasing (or decreasing) acceleration. One example of something moving with constant jerk would be someone in a car starting from rest who presses on the gas pedal harder and harder. The initial change in velocity is small, but it becomes larger and larger. Another example would be a rubber band that is pulled back from someone’s finger and released. When it is fully stretched it tends to restore itself more quickly than it does when it is relaxed. In this case, the jerk would be negative while the acceleration and velocity are both positive. Representative graphs of acceleration and velocity for constant jerk are shown below.

![Graph showing velocity and acceleration for constant jerk](image-url)