Show all work!

The drawing shows two identical systems of objects; each consists of the same three small balls connected by massless rods. In both systems the axis is perpendicular to the page, but it is located at a different place, as shown. The same force of magnitude $F$ is applied to the same ball in each system (see the drawing). The masses of the balls are $m_1 = 9.00$ kg, $m_2 = 6.00$ kg, and $m_3 = 7.00$ kg. The magnitude of the force is $F = 424$ N.

(a) For each of the two systems, determine the moment of inertia about the given axis of rotation.

(b) Calculate the torque (magnitude and direction) acting on each system.

(c) Both systems start from rest, and the direction of the force moves with the system and always points along the 4.00-m rod. What is the angular velocity of each system after 5.00 s?

(a) System $A$: $I = \sum m_r^2 = (9.00 \text{ kg})(0 \text{ m})^2 + (6.00 \text{ kg})(3.00 \text{ m})^2 + (7.00 \text{ kg})(5.00 \text{ m})^2 = 321 \text{ kg m}^2$

System $B$: $I = \sum m_r^2 = (9.00 \text{ kg})(5.00 \text{ m})^2 + (6.00 \text{ kg})(4.00 \text{ m})^2 + (7.00 \text{ kg})(5.00 \text{ m})^2 = 229 \text{ kg m}^2$

(b) $\sum \gamma = \sum 1F1l$

System $A$: $\gamma = (424 \text{ N})(3.00 \text{ m}) = 1272 \text{ Nm clockwise or } -1272 \text{ Nm}$

System $B$: Lever arm = 0 so $\gamma = 0$

(c) $\gamma = \omega \alpha$, so $\gamma = I \omega = I \frac{\Delta \omega}{\Delta t}$, $\omega = \frac{\gamma t}{I}$

System $A$: $\omega = \frac{\gamma t}{I} = (-1272 \text{ Nm})(5.00 \text{ s})/(229 \text{ kg m}^2) = -2.773 \text{ rad/s}$

System $B$: $\omega = 0$, so $\omega = \frac{\gamma t}{I} = 0$
Show all work!

By means of a rope whose mass is negligible, two blocks are suspended over a pulley, as the drawing shows. The pulley can be treated as a uniform solid cylindrical disk. The downward acceleration of the 44.0 kg block is observed to be exactly one-half the acceleration due to gravity. Noting that the tension in the rope is not the same on each side of the pulley, find the mass of the pulley.

Moment of inertia for a solid disk: \( \frac{1}{2} Mr^2 \)

\[
\begin{align*}
\sum F &= T_1 - m_1 g = m_1 a \\
T_1 &= m_1 a + m_2 g \\
\sum F &= T_2 - m_2 g = m_2 (-a) \\
T_2 &= m_2 g - m_2 a
\end{align*}
\]

\[
\sum \gamma = T_1 r - T_2 r = -I \alpha = - \left( \frac{1}{2} Mr^2 \right) \frac{a}{r}
\]

\[
T_1 - T_2 = -\frac{1}{2} m_3 a
\]

\[
m_1 a + m_1 g + m_2 a - m_2 g = -\frac{1}{2} m_3 a \\
a = \frac{1}{2} g:
\]

\[
m_1 \left( \frac{3}{2} g \right) + m_2 \left( -\frac{1}{2} g \right) = -\frac{1}{4} g m_3
\]

\[
m_3 = 2m_2 - 6m_1 = \boxed{22 \text{ kg}}
\]
A tray is moved horizontally back and forth in simple harmonic motion at a frequency of $f = 2.00 \text{ Hz}$. On this tray is an empty cup. Obtain the coefficient of static friction between the tray and the cup, given that the cup begins slipping when the amplitude of the motion is $5.00 \times 10^{-2} \text{ m}$.

**Examine the forces acting on the cup. When it begins to slip, acceleration should be at its maximum.**

\[ \sum F_x = \mu_s N = \text{max}_{(\text{max})} \]

\[ \sum F_y = N - mg = 0 \]

$\rightarrow N = mg$

$\rightarrow \mu_s (mg) = \text{max}_{(\text{max})}$

$\rightarrow \mu_s = \frac{a_{\text{max}}}{g}$

\[ a(t) = -w^2A \cos (wt) \]

$\rightarrow a_{\text{max}} = w^2A \quad \text{(when } \cos(wt) = -1)\]

$\rightarrow \mu_s = \frac{w^2A}{g} = 0.805$
A horizontal spring is lying on a frictionless surface. One end of the spring is attached to a wall, and the other end is connected to a movable object. The spring and object are compressed by 0.065 m, released from rest, and subsequently oscillate back and forth with an angular frequency of 11.3 rad/s. What is the speed of the object at the instant when the spring is stretched by 0.048 m relative to its unstrained length?

\[
\text{speed } = |v(t)| = |\omega A \sin(\omega t)| = \omega A \sin(\omega t)
\]

We need \(\omega t\):

\[
x(t) = A \cos(\omega t)
\]

\[
\Rightarrow \omega t = \cos^{-1}\left(\frac{x(t)}{A}\right) = 137.6^\circ
\]

\[
\Rightarrow |v(t)| = \omega A \sin \left(\cos^{-1}\left(\frac{x(t)}{A}\right)\right)
\]

\[
= (11.3 \text{ rad/s}) (0.065 \text{ m}) \sin(137.6^\circ)
\]

\[
= 0.495 \text{ m/s}
\]

Alternatively, use conservation of energy:

\[
\frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \frac{1}{2} k A^2
\]

\[
\frac{k}{m} x^2 + v^2 = \frac{k}{m} A^2
\]

\[
\omega^2 x^2 + v^2 = \omega^2 A^2
\]

\[
\Rightarrow v = \omega \sqrt{A^2 - x^2}
\]

\[
= (11.3 \text{ rad/s}) \sqrt{(0.065 \text{ m})^2 - (0.048 \text{ m})^2}
\]

\[
= 0.495 \text{ m/s}
\]
A $1.00 \times 10^{-2}$ kg bullet is fired horizontally into a 2.50 kg wooden block attached to one end of a massless horizontal spring ($k = 845 \text{ N/m}$). The other end of the spring is fixed in place, and the spring is unstrained initially. The block rests on a horizontal, frictionless surface. The bullet strikes the block perpendicularly and quickly comes to a halt within it. As a result of this completely inelastic collision, the spring is compressed along its axis and causes the block/bullet to oscillate with an amplitude of 0.200 m. What is the speed of the bullet?

\[ \text{given:} \]
\[ k = 845 \text{ N/m} \]
\[ m = 1.00 \times 10^{-2} \text{ kg} \]
\[ M = 2.50 \text{ kg} \]
\[ A = 0.200 \text{ m} \]

\[ \text{find speed of bullet} \]
\[ \gamma_s \]

\[ \text{conservation of momentum: (inelastic collision)} \]
\[ \gamma_i = \gamma_f \Rightarrow m \gamma_i + M \gamma_i = (m + M) \gamma_f \]
\[ \Rightarrow m \gamma_i = (m + M) \gamma_f \Rightarrow \gamma_i = \frac{(m + M)}{m} \gamma_f \]

\[ \text{After collision mechanical energy is conserved} \]
\[ (\text{because no friction}) \]
\[ T_i + U_i = T_f + U_f \Rightarrow \frac{1}{2} (m + M) \gamma_i^2 + \frac{1}{2} kx_i^2 = \frac{1}{2} (m + M) \gamma_f^2 + \frac{1}{2} kx_f^2 \]
\[ \Rightarrow \frac{1}{2} kx_i^2 = \frac{1}{2} (m + M) \gamma_f^2 \Rightarrow \gamma_f = A \sqrt{\frac{k}{(m + M)}} \]

\[ \text{Note: } \gamma_f \text{ is the speed of the bullet} \]

\[ \Rightarrow \gamma_i = \frac{(m + M)}{m} \left( A \sqrt{\frac{k}{(m + M)}} \right) = \sqrt{\frac{Mk}{m} A^2} \]

\[ \gamma_i = \sqrt{\frac{(1.00 \times 10^{-2}) (845) (1.0 \times 10^{-2} + 2.50)}{(1.0 \times 10^{-2})}} = 1.921 \text{ m/s} \]