A. A disk starting from rest rotates about a fixed axis with a constant angular acceleration. Three identical masses, A, B, and C are on the disk and rotate with the disk. In the spaces below enter A, B, C, or ALL to best fit the statement. Enter ALL if all have the same value. Note: $R_A < R_B < R_C$

1. The mass that will slip first. ________ C ________

2. The mass with the largest moment of inertia ________ C ________

3. The mass with the largest angular acceleration. ________ ALL ________

4. The mass with the largest centripetal acceleration ________ C ________

5. The mass with the largest tangential acceleration ________ C ________

6. The mass with the largest tangential velocity ________ C ________

7. The mass with the smallest kinetic energy ________ A ________

8. The mass with the smallest angular velocity ________ ALL ________

9. The mass with the smallest net torque acting on it ________ A ________

10. The mass with the smallest net force acting on it ________ A ________

B. The figure shows three massless rods all with the same length $\ell$. Objects with different masses are attached to the rods, but the total mass $(3m)$ of the objects is the same for each rod. The figure also shows three different axes of rotation. Axis 1 is located at $x = 0$, axis 2 is located at $x = 1/2 \ell$, and axis 3 is located at $x = \ell$. For each axis, rank the systems (A, B, and C) in terms of their moment of inertia about that axis from greatest to smallest.

**Axis 1:** Biggest ________ C ________ Middle ________ B ________ Smallest ________ A ________

**Axis 2:** Biggest ________ C ________ Middle ________ A ________ Smallest ________ B ________

**Axis 3:** Biggest ________ A ________ Middle ________ B ________ Smallest ________ C ________
Consider a block with mass $m$ attached to a spring with spring constant $k$ set up on a frictionless horizontal surface as in the drawing. The block is pulled a distance $A$ away from its equilibrium position and then released from rest. It is then observed to undergo simple harmonic oscillations.

\[ x = -A \quad x = 0 \quad x = A \]

A. We repeat the same process, but this time use a block of mass $2m$ and the spring with constant $k$.
State whether the following quantities increase, decrease, or remain the same relative to the original experiment.

1. The total energy of the system __________ same
2. The maximum potential energy of the spring __________ same
3. The maximum kinetic energy of the block __________ same
4. The amplitude of the oscillations __________ same
5. The maximum speed of the block __________ decrease
6. The period of oscillations __________ increase
7. The resonant frequency of the spring-mass system __________ decrease

B. We repeat the process once again, but this time we use the block of mass $m$ and a spring with constant $2k$.
State whether the following quantities increase, decrease, or remain the same relative to the original experiment.

1. The total energy of the system __________ increase
2. The maximum potential energy of the spring __________ increase
3. The maximum kinetic energy of the block __________ increase
4. The amplitude of the oscillations __________ same
5. The maximum speed of the block __________ increase
6. The period of oscillations __________ decrease
7. The resonant frequency of the spring-mass system __________ increase

C. We repeat the process one more time. We use the block of mass $m$ and the spring with constant $k$, but this time we pull the block a distance $2A$ from its equilibrium position. State whether the following quantities increase, decrease, or remain the same relative to the original experiment.

1. The total energy of the system __________ increase
2. The maximum potential energy of the spring __________ increase
3. The maximum kinetic energy of the block __________ increase
4. The amplitude of the oscillations __________ increase
5. The maximum speed of the block __________ increase
6. The period of oscillations __________ same
7. The resonant frequency of the spring-mass system __________ same
The figures to the right depict the anatomy of the foot and the forces acting on it as a person stands on the balls of their feet. The force $F_T$ is the force the Achilles tendon exerts on the top of the heel (calcaneus) while $F_B$ is the force the tibia exerts at the ankle joint. $F_N$ is the force exerted by the ground on the ball of the foot. The weight of the foot itself has been neglected due to its relatively small magnitude.

1. A 70 kg person stands so that each foot supports half of their body weight (i.e. $F_N = 0.5 \times 70 \text{ kg}$). Find the magnitude of the force exerted by the Achilles tendon ($F_T$). [Hint: You do not need to know the magnitude of $F_B$.]

   \[
   \text{Sum torques about the ankle: } \sum \tau = F_T d_T \sin 80^\circ - F_N d_N \sin 90^\circ \\
   \text{Equilibrium} \Rightarrow \sum \tau = 0 \Rightarrow - \tau_T = \tau_N \Rightarrow \tau_T = 0 \\
   \Rightarrow F_T = \frac{F_N d_N}{d_T \sin 80^\circ} = \frac{(0.5 \times 70 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (10 \text{ cm})}{(5.6 \text{ cm}) \sin 80^\circ} = 622 \text{ N}
   \]

2. The Achilles tendon of a young athlete has torn and we estimate that at the time of the injury the force on the ball of their foot was about 5700 N. We estimate the length of the tendon to be 20 cm and the cross-sectional area to be about 90 mm$^2$. The average tendon of a young adult has a Young's modulus of about $850 \times 10^6 \text{ N/m}^2$. Assuming the tendon always behaves elastically, how much was the tendon stretched when the injury occurred?

   \[
   \text{For the tendon: } \frac{F_T}{A} = \frac{\gamma AL}{L_0} \Rightarrow AL = \frac{L_0 \cdot F_T}{\gamma A} \\
   \text{For torques as in part 1: } \Rightarrow F_T = \frac{F_N d_N}{d_T \sin 80^\circ} \\
   \Rightarrow AL = \frac{L_0}{\gamma A} \left( \frac{F_N d_N}{d_T \sin 80^\circ} \right) = \frac{(20 \text{ cm}) \times (5700 \text{ N}) 	imes (10 \text{ cm})}{(850 \times 10^6 \text{ N/m}^2) \times (90 \times 10^{-6} \text{ m}^2) \times (5.6 \text{ cm}) \sin 80^\circ} \\
   AL = 2.7 \text{ cm}
   \]

3. Humans are considered to have relatively large heel bones. Within the context of this problem, what is the primary advantage of having a large heel bone? [Hint: Consider how your answers to parts 1 and 2 would change if the heel bone were smaller]

   Large heel bone increases the torque exerted by the Achilles tendon. This allows the ball of the foot to support a greater force before the tendon ruptures. If $d_T$ is greater than $d_N$, $F_N$ can be larger.
A ball with a radius of 0.10 m rolls without slipping at a constant translational speed of 11 m/s along a horizontal table. The ball rolls off the edge and falls a vertical distance of 1.3 m before hitting the floor. How many revolutions does the ball make while it is in the air? Neglect any effects due to air resistance.

1. Take a deep breath. Describe a step-by-step plan to solve this problem. Give meaning to any variables that you use in your description, as well as their value if known. You may use equations to show how the different variables are related to one another. Make sure to point out any key physics concepts and how or where they are to be applied to this problem. You should not solve any equations or calculate any numbers for this part and your plan doesn’t need to. [Hint: If you’re having trouble starting, you may want to begin with a simple sketch of the scenario.]

   - Since the ball is rolling we can find the initial angular velocity by the relation \( \omega = \frac{v_T}{r} \)

   \[ v_T = 11 \text{ m/s} \quad r = 0.10 \text{ m} \]

   - After the ball rolls off the table, the only force acting on the ball is gravity which does not result in any torque on the ball about its center of mass \( \sum \tau = 0 \implies \alpha = 0 \)

   Therefore \( \omega \) is constant and we have \( \Delta \theta = \omega \Delta t \) at \( \Delta t \) time in the air.

   - To find time in the air use kinematics, specifically for the vertical (i.e. \( y \)) component

     \[ a_y = v_{oy} + \frac{1}{2} a_y \Delta t \]

     \[ \Delta y = -1.3 \text{ m} \]

     \[ v_{oy} = 0 \text{ m/s} \]

     \[ a_y = -9.8 \frac{\text{m}}{\text{s}^2} \]

     \[ \Delta t = \frac{\sqrt{2 \Delta y}}{g} \]

   \[ \Delta t = \frac{\sqrt{2(-1.3 \text{ m})}}{9.8 \frac{\text{m}}{\text{s}^2}} \]

2. Execute your plan and solve the problem.

\[ \Delta \theta = \omega \Delta t = \left( \frac{v_T}{r} \right) \sqrt{\frac{-2 \Delta y}{g}} \]

\[ \Delta \theta = \left( \frac{11 \text{ m/s}}{0.10 \text{ m}} \right) \sqrt{\frac{-2(-1.3 \text{ m})}{9.8 \frac{\text{m}}{\text{s}^2}}} \times \frac{1 \text{ rev}}{2 \pi \text{ rad}} = 9.0 \text{ revs.} \]