A related quantity: **Intensity of EM waves** (How bright is the light hitting you?)

Analogous to **sound intensity** (how loud is the sound wave arriving at you)?

Intensity: \( S \) (unit: W/m\(^2\))

\[
S = \frac{\text{Power}}{\text{Area}} = \frac{\text{Total EM energy}}{(\text{Area})\Delta t}
\]

\[
= \frac{(\text{EM energy density})(\text{Volume})}{(\text{Area})\Delta t}
\]

We compute the intensity by finding the energy contained the gray box (of length \( c\Delta t \), and area \( A \)) passing through the yellow frame (area \( A \)) in time \( \Delta t \).

\[
S = \frac{u \cdot (c\Delta t \cdot A)}{A\Delta t} = c \cdot u
\]

\[
\bar{u} = \frac{1}{2} \varepsilon_o E_0^2 = \frac{1}{2\mu_o} B_0^2
\]

\[\rightarrow \bar{S} = c \cdot \bar{u}\]

Average intensity of an electromagnetic wave:

\[
\bar{S} = c \cdot \frac{1}{2} \varepsilon_o E_0^2
\]

and

\[
\bar{S} = c \cdot \frac{1}{2\mu_o} B_0^2
\]
Example: Sunlight enters the top of the earth's atmosphere with an electric field whose RMS value is \( E_{\text{rms}} = 720 \text{ N/C} \). Find

(a) the average intensity of this electromagnetic wave and
(b) the rms value of the sunlight's magnetic field At the Earth, orbiting the Sun at 1.00 a.u.
(c) Repeat (a) and (b) for Venus, which orbits the Sun at a radius of 0.723 a.u.

*** a.u. = astronomical unit.
**Example**: Sunlight enters the top of the earth's atmosphere with an electric field whose RMS value is $E_{\text{rms}} = 720 \, \text{N/C}$. Find

(a) the average intensity of this electromagnetic wave and
(b) the rms value of the sunlight's magnetic field At the Earth, orbiting the Sun at $1.00 \, \text{a.u.}$
(c) Repeat (a) and (b) for Venus, which orbits the Sun at a radius of $0.723 \, \text{a.u.}$

*** a.u. = astronomical unit.
Example: Sunlight enters the top of the earth's atmosphere with an electric field whose RMS value is $E_{\text{rms}} = 720 \text{ N/C}$. Find

(a) the average intensity of this electromagnetic wave and
(b) the rms value of the sunlight's magnetic field At the Earth, orbiting the Sun at 1.00 a.u.
(c) Repeat (a) and (b) for Venus, which orbits the Sun at a radius of 0.723 a.u.

*** a.u. = astronomical unit.

(a) \[ \overline{S} = \frac{1}{2} \varepsilon_0 E_0^2 = \varepsilon_0 E_{\text{rms}}^2 = \left( 3.00 \times 10^8 \text{ m/s} \right) \left( 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \right) \left( 720 \frac{\text{N}}{\text{C}} \right)^2 = 1.38 \times 10^3 \frac{\text{W}}{\text{m}^2} \]

(b) \[ B_0 = \frac{E_0}{c} \rightarrow B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{720 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 2.40 \times 10^{-6} \text{ T} \]

(c) \[ \overline{S}_V = \frac{P_{\text{SUN}}}{4\pi r_V^2} \quad \text{and} \quad \overline{S}_E = \frac{P_{\text{SUN}}}{4\pi r_E^2} \rightarrow \overline{S}_V = \frac{r_E^2}{r_V^2} = \left( \frac{r_V}{r_E} \right)^{-2}, \quad \frac{r_V}{r_E} = \frac{0.723 \text{ a.u.}}{1.00 \text{ a.u.}} = 0.723 \]

\[ \overline{S}_V = \left( \frac{r_V}{r_E} \right)^{-2} \overline{S}_E = \frac{\overline{S}_E}{(r_V / r_E)^2} = 1.38 \times 10^8 \frac{\text{W}}{\text{m}} = 2.63 \times 10^8 \frac{\text{W}}{\text{m}^2} \]

\[ \overline{S} = c \cdot \frac{1}{2\mu_o} B_0^2 = \frac{c}{\mu_o} B_{\text{rms}}^2 \]

\[ \rightarrow B_{\text{rms}} = \sqrt{\frac{\mu_o \overline{S}}{c}} = \sqrt{\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.63 \times 10^3 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}}} = 3.32 \times 10^{-6} \text{ T} \]
24.6 Polarization

In (linearly) polarized light, the electric field oscillates along a single transverse direction.

In unpolarized light, the electric field oscillates along random transverse direction at a given moment in time.
24.6 Polarization

Polarized light may be produced from unpolarized light with the aid of polarizing material.

Average Intensity of light:

\[ S = \frac{1}{2} c\varepsilon_0 E_0^2 \]

Absorption and retransmission by antennae in polarizer

\[ E_0' = E_0 \cos \theta \]

Primed = after polarizer
Unprimed = before polarizer

\[ \bar{S}' = \frac{1}{2} c\varepsilon_0 (E_0')^2 = \frac{1}{2} c\varepsilon_0 (E_0 \cos \theta)^2 \]

\[ \bar{S}' = \left( \frac{1}{2} c\varepsilon_0 E_0^2 \right) \cos^2 \theta = \bar{S} \cos^2 \theta \]

Malus’ Law

For unpolarized incident light

Random Input polarization:

\[ \cos^2 \theta \rightarrow \cos^2 \theta = \frac{1}{2} \]

\[ \bar{S}' = \bar{S} \left( \cos^2 \theta \right) = \frac{1}{2} S \]
24.6 Polarization

MALUS’ LAW (general case)

\[ S = S_0 \cos^2 \theta \]

- \( E_0 \) = polarization before analyzer
- \( E_0 \cos \theta \) = polarization after analyzer
- \( \theta \) = angle between polarization before and after analyzer

The textbook’s notational convention

intensity after analyzer

intensity before analyzer
Example Using Polarizers and Analyzers

What value of $\theta$ should be used so the average intensity of the polarized light reaching the photocell is one-tenth the average intensity of the unpolarized light?

\[
\frac{1}{10} S_o = \left( \frac{1}{2} S_o \right) \cos^2 \theta
\]

\[
\frac{1}{5} = \cos^2 \theta
\]

\[
\cos \theta = \sqrt{\frac{1}{5}}
\]

\[
\theta = 63.4^\circ
\]
24.6 Polarization

THE OCCURANCE OF POLARIZED LIGHT IN NATURE
We will start to treat light in terms of “rays” -- technical for the poetic “beam” of light.

This works as long as the “width” of our rays are significantly larger than the wavelength of the light.
25.1 Wave Fronts and Rays

A hemispherical view of a sound wave emitted by a small pulsating sphere (i.e. a point-like source).

The rays are perpendicular to the wave fronts (locations where the wave is at the same phase angle, e.g. at the maxima or compressions.

At large distances from the source, the wave fronts become less and less curved.

They become plane-waves described by:

$$\psi(x, t) = \psi_0 \sin\left(2\pi ft - \frac{2px}{\lambda}\right)$$
25.2 The Reflection of Light

**LAW OF REFLECTION**

The *incident ray*, the *reflected ray*, and the *normal* to the surface all lie in the same plane, and the angle of incidence equals the angle of reflection.

2 types of reflecting surfaces:

(a) smooth surface

- **Specular reflection**: the reflected rays are parallel to each other.

(b) rough surface

- **Diffuse reflection**

Law of reflection still applies locally, where the normal is the direction perpendicular to the tangent plane.
25.4 Spherical Mirrors

Mirrors cut from segments of a sphere (Spherical mirrors) are used in many optical instruments.

If the inside surface of the spherical mirror is silvered or polished, it is a **concave mirror**.

If the outside surface is silvered or polished, it is a **convex mirror**.

The principal axis of the mirror is a straight line drawn through the center (C) and the midpoint of the mirror (B).

Just as it does for a plane mirror, the Law of reflection still applies locally, where the normal is the direction perpendicular to the tangent plane.
Applications of spherical mirrors

Wide-angle rear-view convex mirror found on many vehicles.

Large radius-of-curvature ($R$) concave mirrors to enlarge nearby object.

e.g. compact mirror