Copper has resistivity of $1.72 \times 10^{-8}\,\Omega \cdot \text{m}$ at 20°C

Example Longer Extension Cords

The instructions for an electric lawn mower suggest that a 20-gauge extension cord can be used for distances up to 35 m, but a thicker 16-gauge cord should be used for longer distances. The cross sectional area of a 20-gauge wire is $5.2 \times 10^{-7} \text{m}^2$, while that of a 16-gauge wire is $1.3 \times 10^{-6} \text{m}^2$. Determine the resistance of (a) 35m of 20-gauge copper wire and (b) 75m of 16-gauge copper wire.

Table 20.1 Resistivities[a] of Various Materials

Material	Resistivity ρ ($\Omega \cdot m$)	Material	Resistivity ρ ($\Omega \cdot m$)
Conductors		***Semiconductors***	
Aluminum	2.82×10^{-8}	Carbon	3.5×10^{-5}
Copper	1.72×10^{-8}	Germanium	0.5^{b}
Gold	2.44×10^{-8}	Silicon	$20\text{–}2300^{b}$
Iron	9.7×10^{-8}	***Insulators***	
Mercury	95.8×10^{-8}	Mica	$10^{11}\text{–}10^{15}$
Nichrome (alloy)	100×10^{-8}	Rubber (hard)	$10^{13}\text{–}10^{16}$
Silver	1.59×10^{-8}	Teflon	10^{16}
Tungsten	5.6×10^{-8}	Wood (maple)	3×10^{10}

[a] The values pertain to temperatures near 20 °C.
[b] Depending on purity.

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(a) $$R_{20} = \rho_{Cu} \frac{L_{20}}{A_{20}} = \frac{(1.72 \times 10^{-8}\,\Omega \cdot \mathrm{m})(35\,\mathrm{m})}{5.2 \times 10^{-7}\,\mathrm{m}^2} = \boxed{1.2\,\Omega}$$

(b) $$R_{16} = \rho_{Cu} \frac{L_{16}}{A_{16}} = \frac{(1.72 \times 10^{-8}\,\Omega \cdot \mathrm{m})(75\,\mathrm{m})}{1.3 \times 10^{-6}\,\mathrm{m}^2} = \boxed{0.99\,\Omega}$$

20.3 Resistance and Resistivity

Over limited temperature ranges, the fractional change in resistivity is proportional to the temperature change. So taking reference resistivity ρ_0 at reference temperature T_0 :

$$\frac{\rho - \rho_o}{\rho_o} = \alpha(T - T_o) \qquad \rho = \rho_o[1+\alpha(T - T_o)] \quad \to R = R_o[1+\alpha(T - T_o)]$$

α: temperature coefficient of resistivity (>0 for metals, <0 for semi-conductors

Neglecting change in dimensions of the resistor

Example: (a) A 34.5m length of copper wire at 20.0°C has a radius of 0.25 mm. If a potential difference of 9.0V is applied across the length of the wire, determine the current in the wire. (b) If the wire is heated to 30.0°C while the 9.0V potential difference is maintained, what is the resulting current in the wire?

Resistivity of Cu $=1.72\times10^{-8}\ \Omega\cdot$m @ 20°C, temperature coefficient: $\alpha = 3.93\times10^{-3}$ C°$^{-1}$

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temperature coefficient of resistivity (>0 for metals)

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Resistivity of Cu =1.72×10^{-8} Ω·m @ 20°C, temperature coefficient: α =3.93×10^{-3} $\mathrm{C^{o\text{-}1}}$

(a) $$R_0 = \rho_0 \frac{L}{A} = \rho_0 \frac{L}{\pi r^2} = (1.72\times10^{-8}\ \Omega\cdot\mathrm{m})\frac{34.5\,\mathrm{m}}{(3.1416)(2.50\times10^{-4}\ \mathrm{m})^2} = 3.02\,\Omega$$

$$I_0 = V/R_0 = (9.0\,\mathrm{V})/(3.02\,\Omega) = \boxed{2.98\,\mathrm{A}}$$

(b) $$R = R_0[1 + \alpha(T - T_0)] = (3.02\,\Omega)[1 + (3.93\times10^{-3}\ \mathrm{C^{o\,\text{-}1}})(30.0^\circ\mathrm{C} - 20.0^\circ\mathrm{C})]$$

$$= (3.02\,\Omega)(1.0393) = 3.14\,\Omega$$

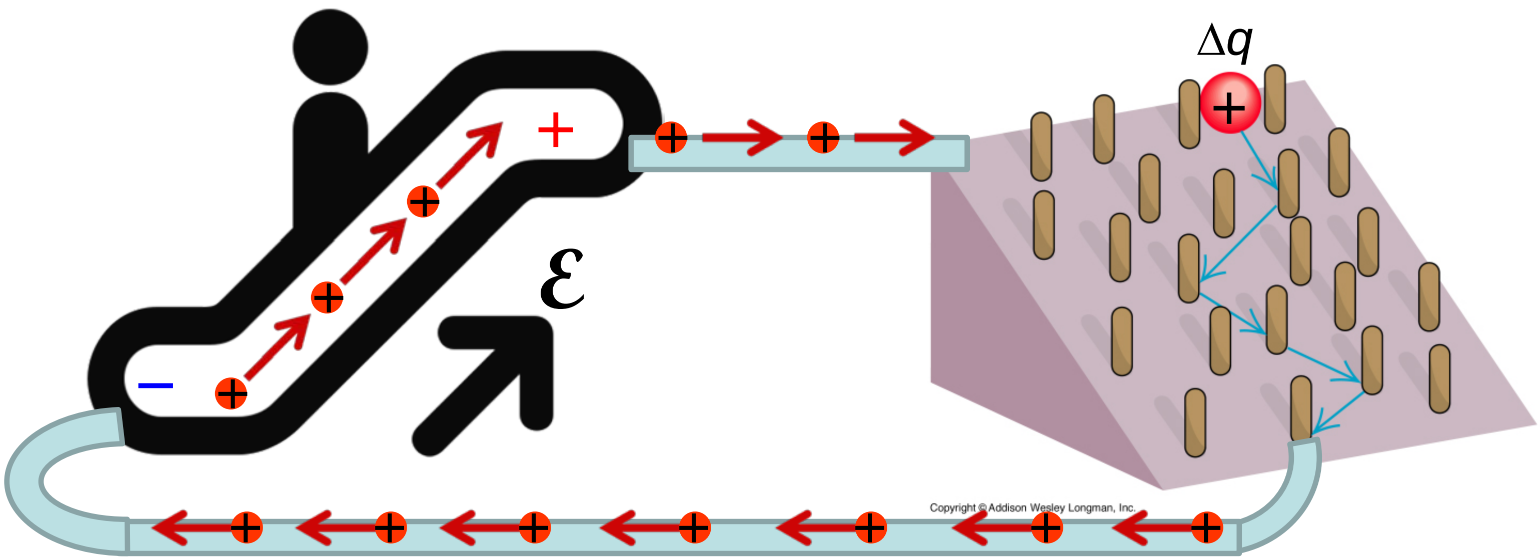
$$I = V/R = (9.0\,\mathrm{V})/(3.14\,\Omega) = \boxed{2.87\,\mathrm{A}}$$

20.4 Electric Power

Electrical Power supplied by a DC source of emf = $\mathcal{E} = V$

Each parcel of charge Δq is raised by a potential difference of V. And gains electric potential energy $\Delta EPE = (\Delta q)\ V$. This is the amount of work ΔW done by the power source on the parcel Δq.

By definition: Power = $\Delta W / \Delta t$



$$P = \frac{\Delta W}{\Delta t} = \frac{(\Delta q)V}{\Delta t} = \frac{\Delta q}{\Delta t}V = IV$$

power

Work/energy

time

The same power generated by the source must then be consumed (or dissipated) in the load resistance:

DC ELECTRIC POWER

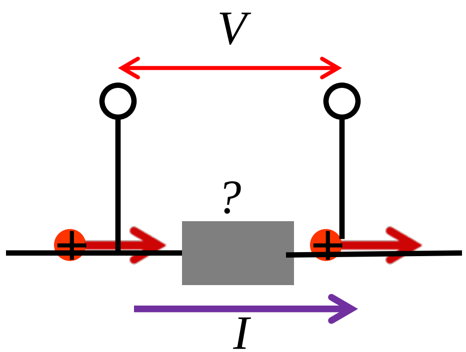
The DC power **generated or consumed** by a two-terminal device, whether it be a DC power source (e.g. a battery) or a resistor (e.g. a light-bulb filament) is in general given by

$$P = IV$$

SI Unit of Power: watt (W=J/s)

In the common usage of this equation, V is the absolute value of the potential difference across the terminals of the device, I is the current through the device,

And P is either the power generated or the power consumed **depending on the context.**



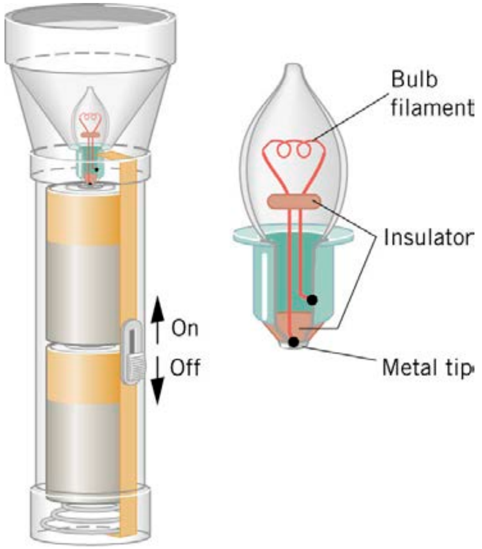
For resistors: (applies to many devices)

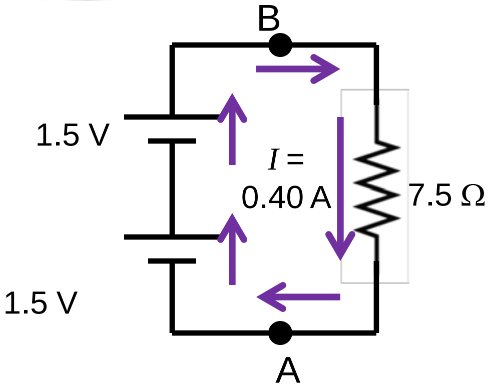
$$P = I(IR) = I^2R \qquad P = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$$

Here P is the power consumed by the resistor

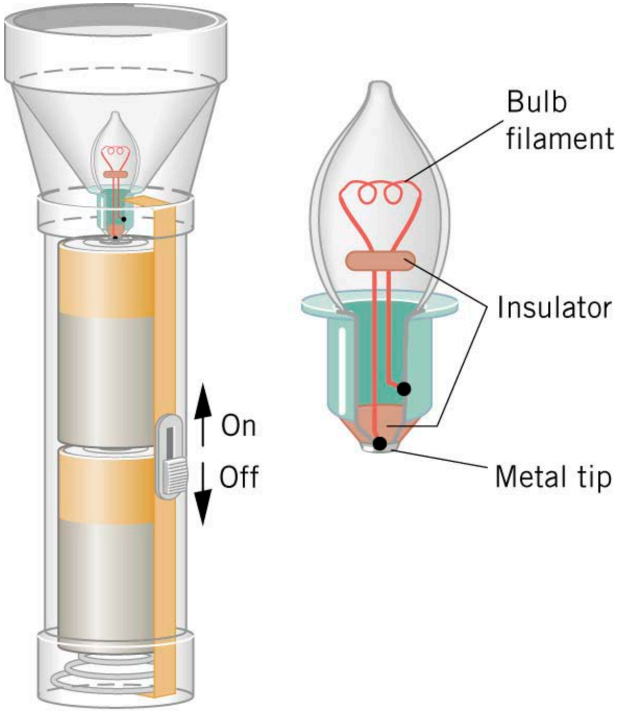
Example The Power and Energy Used in a Flashlight

In the flashlight, the current is 0.40A and the voltage of each of two cells is 1.5 V. Find (a) the power generated by each cell, (b) the power consumed by the bulb, and (c) the energy dissipated in the bulb in 5.5 minutes of operation.





20.4 Electric Power



Example The Power and Energy Used in a Flashlight

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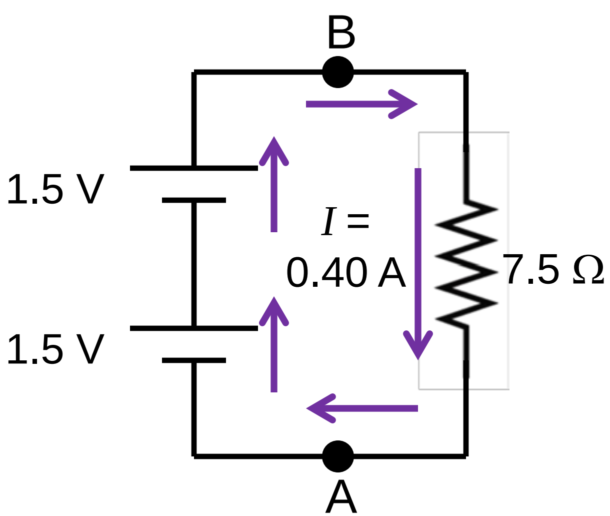
(a) This is a single-loop circuit with no branch points. The 0.40A current flows through the entire circuit, while the potential difference across EACH cell is 1.5 V

$$P_{cell} = I(\Delta V)_{cell} = (0.40\ \text{A})(1.5\ \text{V}) = \boxed{0.60\ \text{W}}$$

(b) Going from point A to B along the left half, the potential increases by $\Delta V = V_B - V_A = 2\times 1.5\text{V} = 3.0\text{V}$ through the two cells.

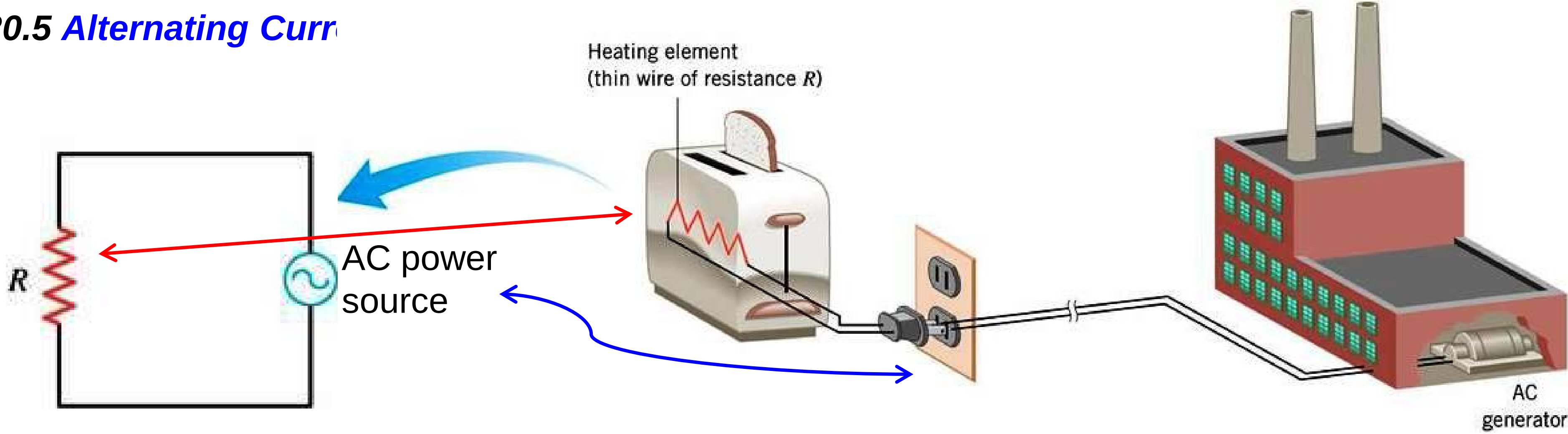


Going from B to A along the right half, the potential drops through the single 7.5W resistor by the same 3.0V

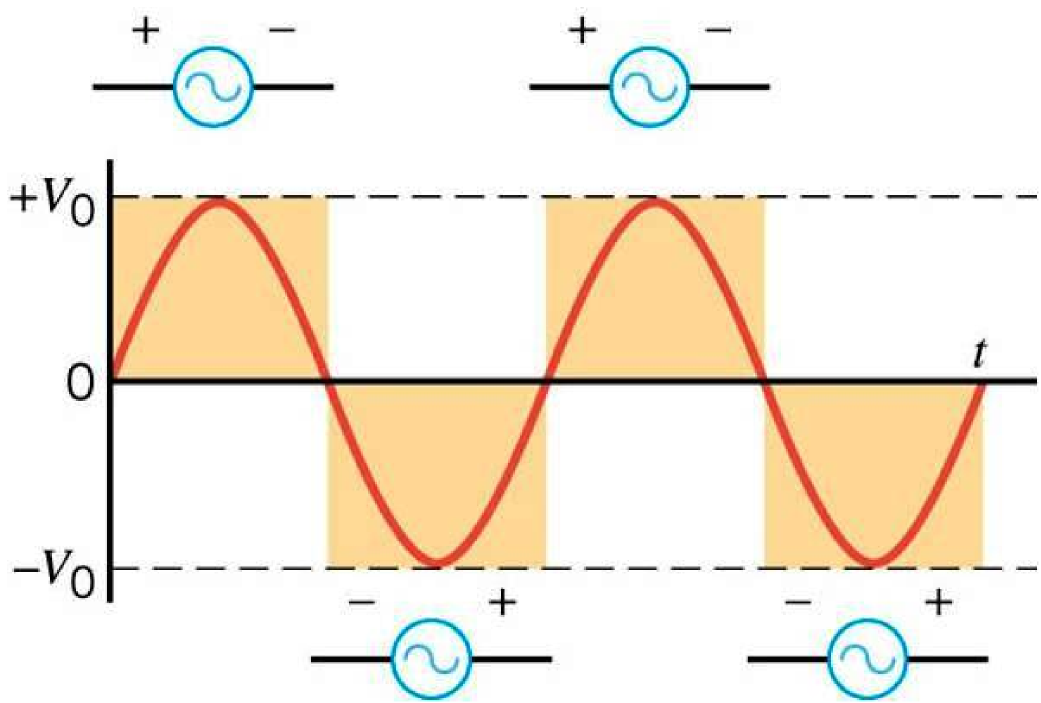
$$P_{bulb} = I(\Delta V)_{bulb} = (0.40\ \text{A})(3.0\ \text{V}) = \boxed{1.20\ \text{W}}$$

(c) Energy consumed = power consumed × time duration

$$E_{bulb} = P_{bulb}\Delta t = (1.2\ \text{W})(330\ \text{s}) = \boxed{4.0\times 10^{2}\ \text{J}}$$



An AC power source (e..g wall socket) provides a voltage (potential difference between the two terminals) that reverses direction periodically. The resulting current in the circuit also alternates direction with the same frequency.



Most common AC power sources provide a voltage that is a sinusoidal function of time:

$$V(t) = V_o \sin(2\pi f t)$$

peak voltage

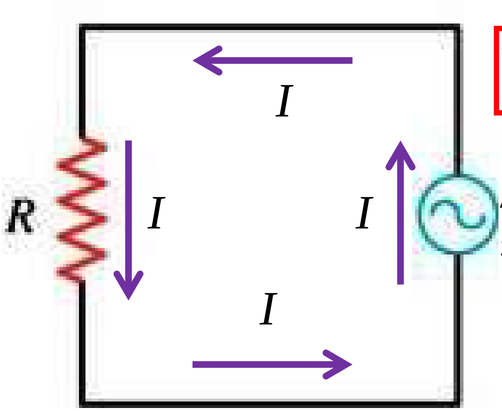
NOT a wave!

U.S. Standard wall socket:
V_0 = 170 V, and f = 60.0 Hz
(120V rms)

Europe
V_0 = 311 V, and f = 50.0 Hz
(220V rms)

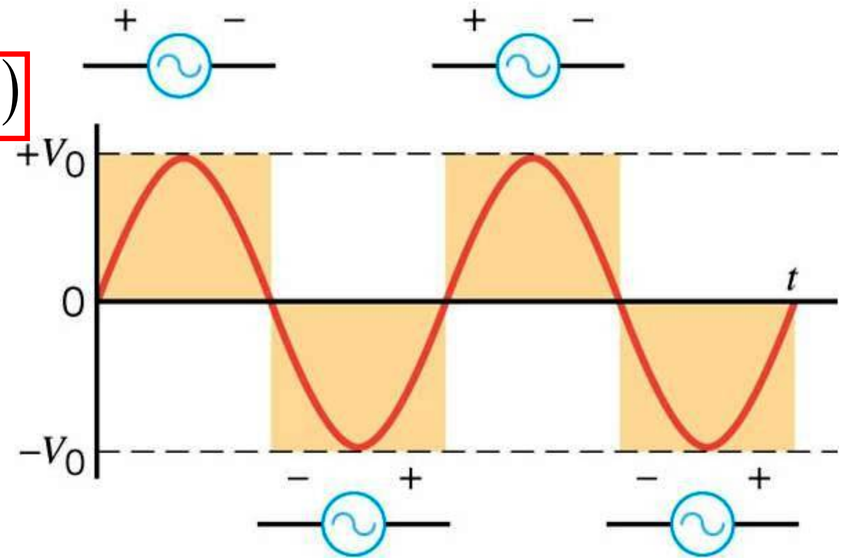
20.5 *Alternating Current*

In circuits that contain only resistance, the current reverses direction each time the polarity of the generator reverses.



$$V(t) = V_o \sin(2\pi ft)$$

AC power source



Ohm's Law still applies (instantaneously)

$$I(t) = \frac{V(t)}{R} = \frac{V_o}{R}\sin(2\pi ft) = I_o \sin(2\pi ft)$$

The sine function can give both a positive or a negative value, depending on the argument

peak current

In these circuits, we label the **current** with a ***presumed direction*** (arbitrary choice) . The **value** of the current I is **positive** when the current actually flows in the **indicated direction**, and **negative** if it flows in the **reverse direction**.

The **instantaneous** power generated by the AC power source is calculated in the usual way: $P = I \cdot V$, ***evaluated instantaneously***

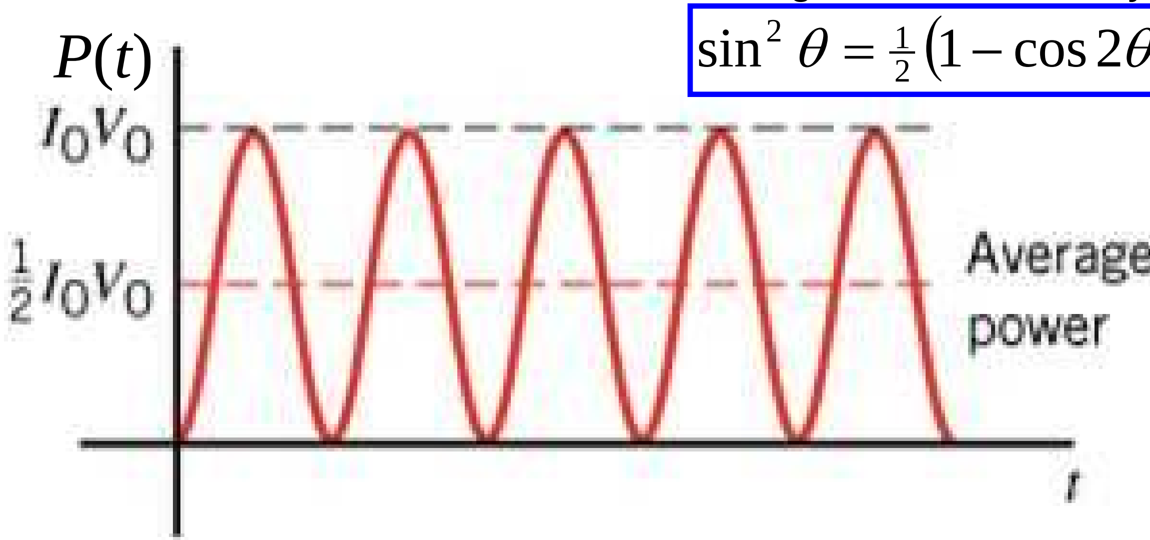
$$I(t) = I_o \sin(2\pi ft) \qquad V(t) = V_o \sin(2\pi ft)$$

sin² function always gives a positive value regardless of argument

$$P(t) = I(t)V(t) = I_o V_o \sin^2(2\pi ft)$$

Trigonometric Identity

$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$



The area between the P-axis and the $P(t)$ curve is **equal** to the area between the $P(t)$ curve and the dashed gray line at the level of $P_0 = I_0 V_0$

The bar/overline means time-average

→ The time-averaged power is ½ the peak value of $P_0 = I_0 V_0$

$$\overline{P} = \frac{I_o V_o}{2} = \left(\frac{I_o}{\sqrt{2}}\right)\left(\frac{V_o}{\sqrt{2}}\right) = I_{rms} V_{rms}$$

rms ?

"rms" stands for "root-mean-squared"
which means: the square-root of the (time) average of the square (of some function of time)

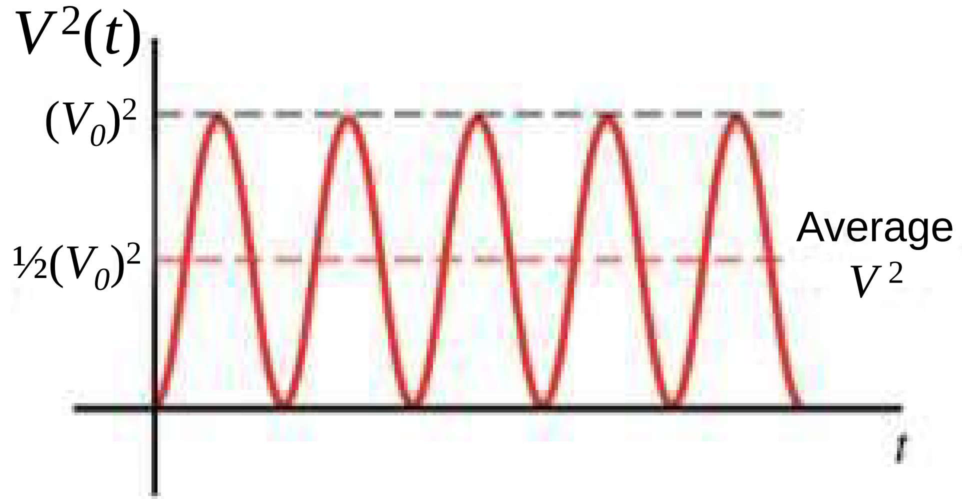
For example, to find V_{rms}, the rms voltage, we plot the square of the voltage function

$$V(t) = V_o \sin(2\pi ft) \rightarrow V^2(t) = (V_o)^2 \sin^2(2\pi ft)$$

Trigonometric Identity

$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

The area between the V^2-axis and the $V^2(t)$ curve is **equal** to the area between the $V^2(t)$ curve and the dashed gray line at the level of $(V_0)^2$



→ The time-averaged V^2 value is ½ the peak value of $(V_0)^2$

$$\rightarrow \overline{V^2} = \tfrac{1}{2}V_0^2$$

$$\rightarrow V_{\mathrm{rms}} = \left(\overline{V^2}\right)^{1/2} = \sqrt{\tfrac{1}{2}V_0^2}$$

$$\Rightarrow \boxed{V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}}}$$

A similar argument gives the rms current

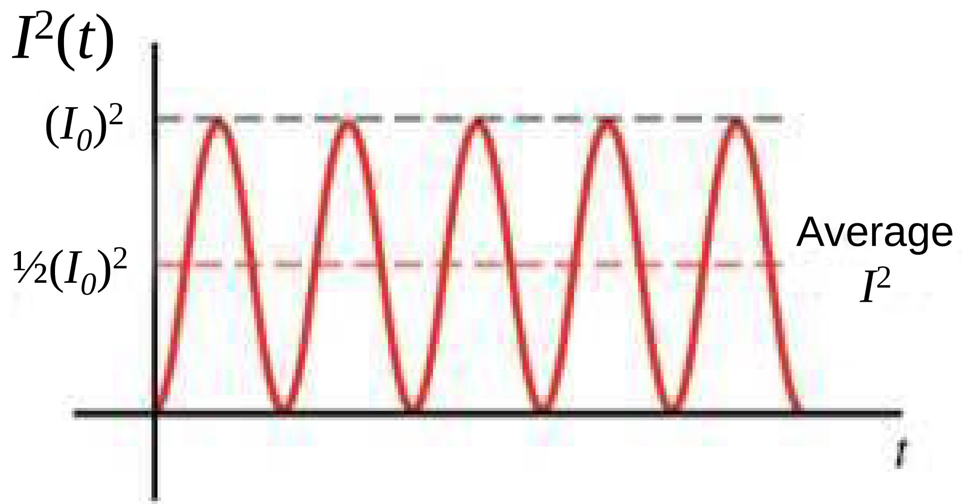
To find I_{rms}, the rms current, we plot the square of the current function

$$I(t) = I_o \sin(2\pi ft) \rightarrow I^2(t) = (I_o)^2 \sin^2(2\pi ft)$$

Trigonometric Identity

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$

The area between the I^2-axis and the $I^2(t)$ curve is **equal** to the area between the $I^2(t)$ curve and the dashed gray line at the level of $(I_0)^2$



→ The time-averaged I^2 value is ½ the peak value of $(I_0)^2$

$$\rightarrow \overline{I^2} = \tfrac{1}{2} I_0^2$$

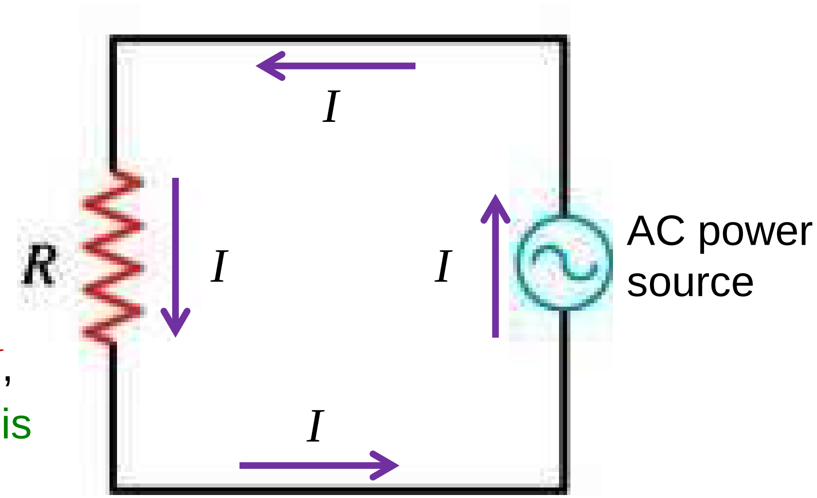
$$\rightarrow I_{\mathrm{rms}} = \left(\overline{I^2}\right)^{1/2} = \sqrt{\tfrac{1}{2} I_0^2}$$

$$\Rightarrow \boxed{I_{\mathrm{rms}} = \frac{I_0}{\sqrt{2}}}$$

For the resistor:

$$V_{\text{rms}} = I_{\text{rms}} R \qquad \overline{P} = V_{\text{rms}} I_{\text{rms}}$$

$$\overline{P} = I_{\text{rms}}^2 R \qquad \overline{P} = \frac{V_{\text{rms}}^2}{R}$$



These equations are identical to the DC case, except that we are using rms values for V and I, the average value for power. The resistance R is the same in both DC and AC cases.

Because they allow us to use the same equations as DC, we use rms values for voltage and current when specifying AC circuits. Also: the ***rms values for voltage and current*** are the DC values that would deliver the same power

Example: An incandescent light bulb is designed to operate directly out of the wall socket in the U.S. When it is warmed up, the filament consume an average of 60 W.

(a) What is the resistance of the filament at operating temperature?
(b) What is the peak current drawn?

$$V_{rms} = 120\text{ V} \ \ (\text{U.S. standard})$$

$$\overline{P} = I_{rms} V_{rms} = \frac{(V_{rms})^2}{R}$$

$$\rightarrow R = \frac{(V_{rms})^2}{\overline{P}} = \frac{(120\text{ J/C})^2}{60\text{ J/s}} = \boxed{240\,\Omega}$$

$$I_0 = \sqrt{2} I_{rms} = \sqrt{2}\frac{V_{rms}}{R} = \frac{\sqrt{2}(120\text{ V})}{240\,\Omega} = \boxed{0.707\text{ A}}$$