The needle of a compass is a permanent magnet that has a north magnetic pole (N) at one end and a south magnetic pole (S) at the other.
(2) Iron/cobalt “bar magnets”

The behavior of magnetic poles is similar to that of like and unlike electric charges.

In Class Demo

Like poles repel

(a)

Unlike poles attract

(b)
21.1 Magnetic Fields

Physics explanation

The Earth is a very large “magnet”

Its “north magnetic pole (N)” — analogous to + charge, near (but not exactly at) the geographical South Pole.

Its “south magnetic pole (S)” — analogous to – charge, near (but not exactly at) the geographical North Pole.

Magnetic Field Lines come out of the north magnetic pole and go into the south magnetic pole in a manner that is analogous to electric field coming out of positive (+) charges and going into (−) charges.

In the full electromagnetic theory by Maxwell (not in the scope of this course)...there is a “duality” between electricity and magnetism.

The magnetic field, \( \vec{B} \), like the electric field, is a vector field — a vector valued function of position.
Surrounding a magnet there is a \textit{magnetic field}. The direction of the magnetic field at any point in space is the direction indicated by the north pole of a small compass needle placed at that point.

\textbf{In Class Demo}

(a) The magnetic field exerts a \textbf{torque} on magnets that tend to make them line up with the field (north pole pointing in the direction of the $B$ field)

(b) The magnet, aligned with the field, will feel a force toward the region of stronger field

(c) When anti-aligned with the field, the magnet will feel a force toward the region of weaker field

http://www.youtube.com/watch?v=7InGRHVRaNw
21.1 Magnetic Fields

The magnetic field lines and pattern of iron filings in the vicinity of a bar magnet and the magnetic field lines in the gap of a horseshoe magnet.

Notice the similarity between the magnetic field lines around the bar magnet and the electric field lines around an electric dipole. They are in fact mathematically the same far away from the source.

The bar magnet is a magnetic dipole: the same number of magnetic field lines come out of the north pole that go into the south pole.

The magnetic monopole (like a magnetic charge), i.e. a pure N or S pole in a single object/particle, is not known to exist, but is not forbidden by Theory.
When a charge is placed in an electric field, it experiences a force, called the **Coulomb Force**, according to

\[ \vec{F} = q\vec{E} \]

*From Chapter 18*
When a charge is placed in a magnetic field, it experiences a force, called the Lorentz Force, according to:

\[ \vec{F}_B = q(\vec{v} \times \vec{B}) \]

“cross” or “vector” product

Right Hand Rule No. 1. Extend the right hand so the fingers point along the direction of the magnetic field and the thumb points along the velocity of the charge. The palm of the hand then faces in the direction of the magnetic force that acts on a positive charge.

The direction of \( \vec{v} \times \vec{B} \) is perpendicular to both \( \vec{v} \) and \( \vec{B} \).

The orientation is given by the right hand rule (RHR - 1 shown here)

\[ |\vec{v} \times \vec{B}| = |\vec{v}||\vec{B}|\sin \theta \]

in this instance, \( \theta \) is the angle (less than 180°) between \( \vec{v} \) and \( \vec{B} \).

If the moving charge is negative, the direction of the force is opposite to that predicted by RHR-1 applied to \( \vec{v} \times \vec{B} \).
21.2 The Force That a Magnetic Field Exerts on a Charge

(a) \( |\vec{v} \times \vec{B}| = 0 \) when \( \vec{v} || \vec{B} \),

(b) maximum when \( \vec{v} \perp \vec{B} \),

(c) in between otherwise

In Class Demo

By RHR - 1: \( \vec{v} \times \vec{B} \) points up
These are electrons so the force points down

http://www.youtube.com/watch?v=7YHwMWcxeX8&feature=related
DEFINITION OF THE MAGNETIC FIELD

The magnitude of the magnetic field at any point in space is defined as:

\[ B = \frac{F}{|q_o| (v \sin \theta)} \]

where the angle \((0<\theta<180^\circ)\) is the angle between the velocity of the charge and the direction of the magnetic field.

**SI Unit of Magnetic Field:** \( \frac{\text{newton} \cdot \text{second}}{\text{coulomb} \cdot \text{meter}} = 1 \text{ tesla (T)} \)

1 gauss = \(10^{-4}\) tesla

Magnetic field at the surface of the Earth \(\sim 0.5\) gauss
Example Magnetic Forces on Charged Particles

A proton in a particle accelerator has a speed of $5.0 \times 10^6$ m/s. The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes an angle of 30.0 degrees with respect to the proton’s velocity (see part (c) of the figure). Find
(a) the magnitude and direction of the force on the proton and
(b) the acceleration of the proton.
(c) What would be the force and acceleration if the particle were an electron?
21.2 The Force That a Magnetic Field Exerts on a Charge

Example Magnetic Forces on Charged Particles

A proton in a particle accelerator has a speed of $5.0 \times 10^6$ m/s. The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes an angle of 30.0 degrees with respect to the proton’s velocity (see part (c) of the figure). Find

(a) the magnitude and direction of the force on the proton and
(b) the acceleration of the proton.
(c) What would be the force and acceleration if the particle were an electron?

\[ \vec{F}_B = q(\vec{v} \times \vec{B}) \]
\[ F = |q|vB \sin \theta \]

(a) \[ F = |q|vB \sin \theta \]
\[ = \left(1.60 \times 10^{-19} \text{C}\right)(5.0 \times 10^6 \text{ m/s})(0.40 \text{ T})\sin(30.0^\circ) \]
\[ = 1.6 \times 10^{-13} \text{ N} \]

(b) \[ a = \frac{F}{m_p} = \frac{1.6 \times 10^{-13} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 9.6 \times 10^{13} \text{ m/s}^2 \text{ upward} \]

(c) Magnitude of the force is the same, but direction is opposite. The mass is \(~1800\) times smaller
\[ a = \frac{F}{m_e} = \frac{1.6 \times 10^{-13} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = 1.8 \times 10^{17} \text{ m/s}^2 \text{ downward} \]
A charged particle moving perpendicular to a uniform magnetic field executes uniform circular motion.

This is because the force is always perpendicular to the velocity and does not change the speed.

The magnetic force always remains perpendicular to the velocity and is directed toward the center of the circular path.

The cyclotron frequency is given by:

\[ F_c = m \frac{v^2}{r} \]

\[ qvB = m \frac{v^2}{r} \]

\[ r = \frac{mv}{qB}, \text{ and } \omega = \frac{v}{r} = \frac{qB}{m} \]
21.4 The Mass Spectrometer

**Application:** circular motion of moving ions in a uniform magnetic field:

The mass spectrometer

\[ r = \frac{mv}{qB} = \frac{mv}{eB}, \]

\[ \frac{1}{2} mv^2 = eV \]

magnitude of electron charge

KE=PE

\[ m = \left( \frac{er^2}{2V} \right)B^2 \]

The mass spectrum of naturally occurring neon, showing three isotopes.
**Example**: A singly charged positive ion has a mass of $2.5 \times 10^{-26}$ kg. After being accelerated through a potential difference of 250 V, the ion enters a magnetic field of 0.5 T, in a direction perpendicular to the field. Calculate the radius of the path of the ion in the field.
Example: A singly charged positive ion has a mass of $2.5 \times 10^{-26}$ kg. After being accelerated through a potential difference of 250 V, the ion enters a magnetic field of 0.5 T, in a direction perpendicular to the field. Calculate the radius of the path of the ion in the field.

$q = 1.6 \times 10^{-19}$ C

$m = 2.5 \times 10^{-26}$ kg

$\Delta V = 250$ V

$B = 0.5$ T

$r = ?$

$F_B = F_c \quad qvB = \frac{mv^2}{r} \quad r = \frac{mv}{qB}$

We need to solve for the velocity!

$\Delta V = \frac{W}{q} = \frac{\Delta K}{q} = \frac{\frac{1}{2}mv^2}{q}$

$v = \sqrt{\frac{2\Delta V q}{m}} = \sqrt{\frac{2(250)(1.6 \times 10^{-19})}{2.5 \times 10^{-26}}} = 5.66 \times 10^4$ m/s

$r = \frac{(2.5 \times 10^{-26})(56,568)}{(1.6 \times 10^{-19})(0.5)} = 0.0177$ m
**Example: Velocity Selector**

A velocity selector is a device for measuring the velocity of a charged particle. The device operates by applying electric and magnetic forces to the particle in such a way that these forces balance.

Given $B$ and $q$, wow should an electric field be applied so that the force it applies to the particle can balance the magnetic force?

**Solution:** by RHR-1: the velocity is to the right (thumb) and the magnetic field (finger) is into the page (“x” marks the tail of an arrow), so the magnetic force on a **positive charge** is **upward** (palm up). Here $\theta = 90^\circ \rightarrow F_B = qvB$

We therefore need the electric force $F_E$ that points down that has the same magnitude: for a positive charge the electric field then needs to point in the same direction as the desired force, and $F_E = qE$.

We want $F_E = F_B \rightarrow qE = qvB$

$\Rightarrow E = vB$ (and pointing downward)
21.5 The Force on a Current in a Magnetic Field

**Magnetic force on a current**

The magnetic force on the moving charges pushes the wire to the right.

Since a current consists of moving charges then a current is subject to the Lorentz force.

**Magnetic force on a straight current segment of length L**

\[ F = qvB \sin \theta \]

\[ F = \left( \frac{\Delta q}{\Delta t} \right) \left( v \Delta t \right) B \sin \theta \]

Here \( \theta \) is the angle between the direction of the current and the magnetic field.

\[ F = ILB \sin \theta \]
21.5 The Force on a Current in a Magnetic Field

Example: A wire carries a current of 22.0 A from west to east. Assume that at this location the magnetic field of Earth is horizontal and directed from south to north and that it has a magnitude of $B=0.500 \times 10^{-4}$ T.

(a) Find the magnitude and direction of the magnetic force on a 36.0 m length of wire.
(b) Calculate the gravitational force on the same length of wire if it’s made of copper and has a cross-sectional area of $2.50 \times 10^{-6}$ m$^2$. 
Example: A wire carries a current of 22.0 A from west to east. Assume that at this location the magnetic field of Earth is horizontal and directed from south to north and that it has a magnitude of \( B = 0.500 \times 10^{-4} \) T.
(a) Find the magnitude and direction of the magnetic force on a 36.0 m length of wire.
(b) Calculate the gravitational force on the same length of wire if it’s made of copper and has a cross-sectional area of \( 2.50 \times 10^{-6} \) m\(^2\).

Solution:
(a) By RHR - 1 the force is pointing upward
\[
F_B = ILB \sin \theta
\]
where \( \theta \) is the angle between the direction of the current segment and the magnetic field
Here \( \theta = 90^\circ \rightarrow F_B = ILB = (22.0 \, \text{A})(36.0 \, \text{m})(5.00 \times 10^{-5} \, \text{T}) = 3.96 \times 10^{-2} \, \text{N (up)} \)

(b) Density of copper is \( \rho = 8.94 \times 10^3 \, \text{kg/m}^3 \)
Mass of wire: \( M = \rho (volume) = \rho AL \)
\[
M = (8.94 \times 10^3 \, \text{kg/m}^3)(2.50 \times 10^{-6} \, \text{m}^2)(36.0 \, \text{m}) = 0.805 \, \text{kg}
\]
\[
F_G = Mg = (0.805 \, \text{kg})(9.81 \, \text{m/s}^2) = 7.89 \, \text{N}
\]
\[
\frac{F_G}{F_B} = 199
\]
21.6 The Torque on a Current-Carrying Coil

Put a rectangular loop of current \( I \) and length (height) \( L \), and width \( w \) in a uniform magnetic field \( B \). The loop is mounted such that it is free to rotate about a vertical axis through its center. We will consider the forces on each segment and the resulting torque from each.

Using RHR-1: The force on segment 3 points down, and that on segment 4 points up. \( \vec{F}_3 \) and \( \vec{F}_4 \) are also equal in magnitude and cancel one another.

The magnitudes \( F_3 = F_4 = IwB\sin(90° - \phi) = IwB\cos\phi \) also change with the rotation angle \( \phi \).

But both \( \vec{F}_3 \) and \( \vec{F}_4 \) are directed parallel to the axis, and results in no torque.

![Diagram of a current-carrying coil](image)
Looking at segments 1 and 2 which have the current running vertically.

By RHR-1, force $\vec{F}_1$ on segment 1 (current up) points into the page, for all values of $\phi$. Also by RHR-1, force $\vec{F}_2$ on segment 1 (current down) points out of the page. They cancel each other to yield no net force on the loop.

However, $\vec{F}_1$ and $\vec{F}_2$ both tend to turn the loop in the **clockwise** sense (as seen in the **top view**). The torques from the two forces are each

$$\tau_{1,2} = (F_{1,2}) \left( \frac{w}{2} \right) \sin \phi$$

$$F = ILB \sin \theta = ILB$$

since $\theta = 90^\circ$

$$\tau = \tau_1 + \tau_2 = Fw \sin \phi$$

$\phi$ is the angle between the “normal” to the loop and the magnetic field.
21.6 The Torque on a Current-Carrying Coil

\[ \tau = \tau_1 + \tau_2 = Fw \sin \phi \]

The torque \( \tau \) is maximum when the normal of the loop is perpendicular to the magnetic field, and zero when the normal is parallel to the field.

The torque tends to cause the loop normal to become aligned to the field, just like on a bar magnet. **Current loop = magnetic dipole**

\[ F = ILB \sin \theta = ILB \]

since \( \theta = 90^\circ \)

Net torque \( \tau = ILB (w \sin \phi) = IAB \sin \phi \)

**Top view**

\[ A = Lw = \text{area of loop} \]

\[ \tau = NIA B \sin \phi \]

magnetic moment \( m \)

number of turns of wire
Example The Torque Exerted on a Current-Carrying Coil

A coil of wire has an area of $2.0 \times 10^{-4} \text{m}^2$, consists of 100 loops or turns, and contains a current of 0.045 A. The coil is placed in a uniform magnetic field of magnitude 0.15 T.

(a) Determine the magnetic moment of the coil.
(b) Find the maximum torque that the magnetic field can exert on the coil.
21.6 The Torque on a Current-Carrying Coil

Example The Torque Exerted on a Current-Carrying Coil

A coil of wire has an area of $2.0 \times 10^{-4} \text{m}^2$, consists of 100 loops or turns, and contains a current of 0.045 A. The coil is placed in a uniform magnetic field of magnitude 0.15 T.

(a) Determine the magnetic moment of the coil.

(b) Find the maximum torque that the magnetic field can exert on the coil.

(a) $m = NIA = (100)(0.045 \text{ A})(2.0 \times 10^{-4} \text{ m}^2) = 9.0 \times 10^{-4} \text{ A} \cdot \text{m}^2$

(b) $\tau = NIA B \sin \phi = (9.0 \times 10^{-4} \text{ A} \cdot \text{m}^2)(0.15 \text{ T})\sin 90^\circ = 1.4 \times 10^{-4} \text{ N} \cdot \text{m}$