Electromagnetic (EM) waves also can exhibit a Doppler effect:

1. **Increase** in observed frequency for source and observer **approaching** one another
2. **Decrease** in observed frequency for source and observer **receding** from one another

But Doppler Shift of EM waves differs from that of sound for two reasons:

a) Sound waves travels in a medium, whereas electromagnetic waves can travel in a vacuum (and in some transparent medium)

b) For sound, it is the motion relative to the medium that is important. For electromagnetic waves, only the relative motion of the source and observer is important.

For sound:

\[
\frac{f'}{f} = \left(1 \pm \frac{v}{v_s}\right) \left(1 \pm \frac{v}{v}\right)
\]

Not for light:

\[
f_o = f_s \left(1 \pm \frac{v_{rel}}{c}\right)
\]

if \(v_{rel} \ll c\)
**Example** Radar Guns and Speed Traps. The radar gun of a police car emits an electromagnetic wave with a frequency of $8.0 \times 10^9$ Hz. The approach is essentially **head on**. The wave from the gun **reflects from the speeding car** and returns to the police car, where on-board equipment measures its frequency to be greater than the emitted wave by 2100 Hz. Find the **speed of the car** with respect to the highway. **The police car is parked.**
Example  Radar Guns and Speed Traps.  The radar gun of a police car emits an electromagnetic wave with a frequency of $8.0 \times 10^9$ Hz.  The approach is essentially head on.  The wave from the gun reflects from the speeding car and returns to the police car, where on-board equipment measures its frequency to be greater than the emitted wave by 2100 Hz.  Find the speed of the car with respect to the highway.  The police car is parked.
Example  Radar Guns and Speed Traps

The radar gun of a police car emits an electromagnetic wave with a frequency of 8.0×10⁹ Hz. The approach is essentially **head on**. The wave from the gun reflects from the speeding car and returns to the police car, where on-board equipment measures its frequency to be greater than the emitted wave by 2100 Hz. Find the speed of the car with respect to the highway. The police car is parked.

\[
\Delta f = f'_o - f_s
\]

\[
= f_o \left(1+\frac{v}{c}\right) - f_s = f_s \left[\left(1+\frac{v}{c}\right)\left(1+\frac{v}{c}\right)\right] - f_s = f_s \left(1 + \frac{2v}{c} + \frac{v^2}{c^2}\right) - f_s
\]

\[
= f_s \left[\left(1 + \frac{2v}{c} + \frac{v^2}{c^2}\right) - 1\right] = f_s \left(\frac{2v}{c} + \frac{v^2}{c^2}\right)
\]

\[
\frac{v}{c} \ll 1 \rightarrow \Delta f \approx f_s \frac{2v}{c}
\]

\[
v \approx c \frac{\Delta f}{2f_s} = \left(3.0 \times 10^8 \text{ m/s}\right) \frac{2100 \text{ Hz}}{2(8.0 \times 10^9 \text{ Hz})} = 39 \text{ m/s}
\]
Electrical Energy stored in a capacitor

can be thought of in two different ways:

(1) Electrical potential energy between the excess $+q$ and $-q$ charges spread evenly separated by distance $d$, RELATIVE to the situation where the two plates are neutral.

(2) Potential energy **stored in the electric field** (which exists only between the plates, and is zero outside!!!)

\[
\text{Energy} = \frac{1}{2} CV^2 \quad V = Ed
\]

\[
\text{Energy} = \frac{1}{2} \left( \frac{\kappa \varepsilon_0 A}{d} \right) (Ed)^2 = \left( \frac{1}{2} \kappa \varepsilon_0 E^2 \right) (Ad) = u(\text{volume})
\]

\[
C = \frac{k \varepsilon_0 A}{d}
\]

\[
u = \text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \kappa \varepsilon_0 E^2
\]

\[
= \frac{1}{2} \varepsilon_0 E^2 \quad \text{in vacuum}
\]

Electric field carries energy of “density”
22.8 Self Inductance

Magnetic Energy: (1) Inductance

Take an IDEAL solenoid of $N$ turns, cross-sectional area $A$, length $h$ and carrying current $I$.

We now want to increase the current by $\Delta I$ over time interval $\Delta t$.

This will cause the magnetic field generated by the solenoid to change, and then induce a back-EMF that tends to prevent the increase in current (Lenz’s Law!).

$$\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} = -NA \frac{\Delta B}{\Delta t}$$

$$B = \mu_0 nI = \frac{\mu_0 NI}{h}$$

$$\mathcal{E} = -NA \frac{1}{\Delta t} \Delta \left( \frac{\mu_0 NI}{h} \right) = -NA \left( \frac{\mu_0 N}{h} \right) \frac{\Delta I}{\Delta t}$$

$$\mathcal{E} = -L \frac{\Delta I}{\Delta t}, \quad L = \frac{\mu_0 N^2 A}{h}$$

**SI Unit of $L$**

$1\text{ V} \cdot \text{s}/\text{A} = 1\text{ H}$ (Henry)

Where $L$ is defined to be the (self-) inductance of the solenoid
(2) The energy stored in an inductor

The work required to increase the current from \( I \) by a small \( \Delta I \) is given by:

\[
\Delta W = -\mathcal{E} \Delta q = -\mathcal{E}(I \Delta t)
\]

\[
\Delta W = \left( L \frac{\Delta I}{\Delta t} \right)(I \Delta t) = (LI) \Delta I
\]

\[
\frac{\Delta W}{\Delta I} = LI
\]

Total magnetic potential energy (MPE) stored is then equal to the total work required to ramp up the current from 0 to I, which is given by the triangular area under the line

\[
\Delta W / \Delta I = LI
\]

\[
MPE = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{h} I^2
\]

\[
= \frac{1}{2} \mu_0 \left( \frac{N^2 I^2}{h} \right) Ah = \left( \frac{1}{2 \mu_0} B^2 \right) (\text{Volume})
\]
24.4 The Energy Carried by Electromagnetic Waves

From Ch. 20 electric energy density

$$u_E = \frac{\text{Total } EPE}{\text{Volume}} = \frac{1}{2} \varepsilon_o E^2$$

From Ch. 22 magnetic energy density

$$u_B = \frac{\text{Total } MPE}{\text{Volume}} = \frac{1}{2 \mu_o} B^2$$

The total energy density carried by an electromagnetic wave (which is itself a function of location and time)

$$u(x, t) = \frac{\text{Total } (EPE + MPE)}{\text{Volume}} = \frac{1}{2} \varepsilon_o [E(x, t)]^2 + \frac{1}{2 \mu_o} [B(x, t)]^2$$

$$u(x, t) = \frac{1}{2} \varepsilon_o E_o^2 \sin^2 \left(2\pi ft + \frac{2\pi x}{\lambda}\right) + \frac{1}{2 \mu_o} B_o^2 \sin^2 \left(2\pi ft + \frac{2\pi x}{\lambda}\right)$$

$$\sin^2 \left(2\pi ft + \frac{2\pi x}{\lambda}\right) = \frac{1}{2} \rightarrow \bar{u} = \frac{1}{2} \varepsilon_o \left(\frac{1}{2} E_o^2\right) + \frac{1}{2 \mu_o} \left(\frac{1}{2} B_o^2\right)$$

But $$B_0 = \frac{E_0}{c} = \sqrt{\varepsilon_o \mu_o} E_0 \rightarrow B_0^2 = \varepsilon_o \mu_o E_0^2 \rightarrow \frac{B_0^2}{\mu_o} = \varepsilon_o E_0^2$$

Then

$$\bar{u} = \frac{1}{2} \varepsilon_o E_0^2$$

and

$$\bar{u} = \frac{1}{2 \mu_o} B_0^2$$
24.4 The Energy Carried by Electromagnetic Waves

A related quantity: **Intensity of EM waves** (How bright is the light hitting you?)
Analogous to **sound intensity** (how loud is the sound wave arriving at you)?

Intensity: \( S \) (unit: \( W/m^2 \))

\[
S = \frac{\text{Power}}{\text{Area}} = \frac{\text{Total EM energy}}{(\text{Area})\Delta t} = \frac{(\text{EM energy density})(\text{Volume})}{(\text{Area})\Delta t}
\]

We compute the intensity by finding the energy contained the gray box (of length \( c\Delta t \), and area \( A \)) passing through the yellow frame (area \( A \)) in time \( \Delta t \).

\[
S = \frac{u \cdot (c\Delta t \cdot A)}{A\Delta t} = c \cdot u
\]

\[\rightarrow \bar{S} = c \cdot \bar{u}\]

**Average intensity of an electromagnetic wave:**

\[
\bar{S} = c \cdot \frac{1}{2} \varepsilon_o E_o^2
\]

and

\[
\bar{S} = c \cdot \frac{1}{2\mu_o} B_o^2
\]
Example: Sunlight enters the top of the earth's atmosphere with an electric field whose RMS value is $E_{\text{rms}} = 720 \text{ N/C}$. Find

(a) the average intensity of this electromagnetic wave and
(b) the rms value of the sunlight's magnetic field At the Earth, orbiting the Sun at 1.00 a.u.
(c) Repeat (a) and (b) for Venus, which orbits the Sun at a radius of 0.723 a.u.

*** a.u. = astronomical unit.
**Example:** Sunlight enters the top of the earth's atmosphere with an electric field whose RMS value is $E_{rms} = 720 \text{ N/C}$. Find

(a) the average intensity of this electromagnetic wave and
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(c) Repeat (a) and (b) for Venus, which orbits the Sun at a radius of 0.723 a.u.

*** a.u. = astronomical unit.

(a) \[ \overline{S} = \frac{1}{2} c \varepsilon_o E_0^2 = c \varepsilon_o E_{rms}^2 = \left(3.00 \times 10^8 \text{ m/s} \right) \left(8.85 \times 10^{-12} \text{ C}^2 \text{ N} \cdot \text{m}^2 \right) \left(720 \frac{\text{N}}{\text{C}} \right)^2 = 1.38 \times 10^3 \frac{\text{W}}{\text{m}^2} \]

(b) \[ B_0 = \frac{E_0}{c} \rightarrow B_{rms} = \frac{E_{rms}}{c} = \frac{720 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 2.40 \times 10^{-6} \text{ T} \]

(c) \[ \overline{S}_V = \frac{P_{\text{SUN}}}{4 \pi r_v^2} \quad \text{and} \quad \overline{S}_E = \frac{P_{\text{SUN}}}{4 \pi r_E^2} \rightarrow \overline{S}_V = \overline{S}_E = \frac{r_E^2}{r_V^2} = \left(\frac{r_V}{r_E}\right)^{-2}, \quad \frac{r_V}{r_E} = \frac{0.723 \text{ a.u.}}{1.00 \text{ a.u.}} = 0.723 \]

\[ \overline{S}_V = \left(\frac{r_V}{r_E}\right)^{-2} \overline{S}_E = \frac{\overline{S}_E}{\left(\frac{r_V}{r_E}\right)^2} = \frac{1.38 \times 10^8 \text{ W/m}}{(0.723)^2} = 2.63 \times 10^8 \frac{\text{W}}{\text{m}^2} \]

\[ \overline{S} = c \cdot \frac{1}{2 \mu_o} B_0^2 = \frac{c}{\mu_o} B_{rms}^2 \]

\[ \rightarrow B_{rms} = \sqrt{\frac{\mu_o \overline{S}}{c}} = \sqrt{\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.63 \times 10^8 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}}} = 3.32 \times 10^{-6} \text{ T} \]
In (linearly) polarized light, the electric field oscillates along a single transverse direction.

In unpolarized light, the electric field oscillates along random transverse direction at a given moment in time.
**24.6 Polarization**

Polarized light may be produced from unpolarized light with the aid of polarizing material.

**Average Intensity of light:**

\[ \overline{S} = \frac{1}{2} c \varepsilon_0 E_0^2 \]

**Absorption and retransmission by antennae in polarizer**

\[ E_0' = E_0 \cos \theta \]

Primed = after polarizer
Unprimed = before polarizer

**Malus’ Law**

For **unpolarized incident light**

Random Input polarization:

\[ \cos^2 \theta \rightarrow \overline{\cos^2 \theta} = \frac{1}{2} \]

\[ \overline{S'} = \overline{S \left( \cos^2 \theta \right)} = \frac{1}{2} \overline{S} \]

For unpolarized incident light

Random Input polarization:

\[ \cos^2 \theta \rightarrow \overline{\cos^2 \theta} = \frac{1}{2} \]

\[ \overline{S'} = \overline{S \left( \cos^2 \theta \right)} = \frac{1}{2} \overline{S} \]
MALUS’ LAW (general case)

\[ \theta = \text{angle between polarization before and after analyzer} \]

\[ \overline{S} = \overline{S}_0 \cos^2 \theta \]

The textbook’s notational convention

Intensity before analyzer

Intensity after analyzer
Example Using Polarizers and Analyzers

What value of $\theta$ should be used so the average intensity of the polarized light reaching the photocell is one-tenth the average intensity of the unpolarized light?

\[
\frac{1}{10} \overline{S_o} = \left( \frac{1}{2} \overline{S_o} \right) \cos^2 \theta
\]

\[
\frac{1}{5} = \cos^2 \theta
\]

\[
\cos \theta = \sqrt{\frac{1}{5}}
\]

$\theta = 63.4^\circ$
THE OCCURANCE OF POLARIZED LIGHT IN NATURE
Chapter 25

The Reflection of Light: Mirrors

We will start to treat light in terms of “rays” -- technical for the poetic “beam” of light.

This works as long as the “width” of our rays are significantly larger than the wavelength of the light.
25.1 Wave Fronts and Rays

A hemispherical view of a sound wave emitted by a small pulsating sphere (i.e. a point-like source).

The rays are perpendicular to the wave fronts (locations where the wave is at the same phase angle, e.g. at the maxima or compressions.

At large distances from the source, the wave fronts become less and less curved.

They become plane-waves described by:

\[ \psi(x, t) = \psi_0 \sin \left(2\pi ft - \frac{2\pi x}{\lambda}\right) \]
25.2 *The Reflection of Light*

**LAW OF REFLECTION**

The *incident ray*, the *reflected ray*, and the *normal* to the surface all lie in the **same plane**, and the **angle of incidence equals the angle of reflection**.

2 types of reflecting surfaces:

(a) **smooth surface**

![Specular reflection](image)

*Specular reflection*: the reflected rays are parallel to each other.

(b) **rough surface**

![Diffuse reflection](image)

*Diffuse reflection*

Law of reflection still applies locally, where the normal is the direction perpendicular to the tangent plane.
25.4 Spherical Mirrors

Mirrors cut from segments of a sphere (Spherical mirrors) are used in many optical instruments.

If the **inside surface** of the spherical mirror is silvered or polished, it is a **concave mirror**.

If the **outside surface** is silvered or polished, it is a **convex mirror**.

The **principal axis** of the mirror is a straight line drawn through the center (C) and the midpoint of the mirror (B).

Just as it does for a plane mirror, the Law of reflection still applies locally, where the normal is the direction perpendicular to the tangent plane.
Applications of spherical mirrors

Large radius-of-curvature ($R$) concave mirrors to enlarge nearby object.

- e.g. compact mirror

Wide-angle rear-view convex mirror found on many vehicles.

[Image of a compact mirror and a rearview mirror.]
Light rays near and parallel to the principal axis are reflected from the concave mirror and converge at the focal point.

The focal length, \( f \), is the distance between the focal point and the middle of the mirror.

The focal point of a spherical concave mirror is (approximately) halfway between the center of curvature of the mirror (\( C \)) and the middle of the mirror at \( B \).

\[
f = \frac{1}{2} R
\]
25.4 Spherical Mirrors

Rays that lie close to the principal axis are called **paraxial rays**.

Rays that are far from the principal axis do not converge to a single point.

The effect is called **spherical aberration**.

When paraxial light rays that are parallel to the principal axis strike a convex mirror, the rays appear to originate from the **focal point behind the mirror**.

Since the light rays do not really converge to this point, it is a **virtual focus**.

$$f = -\frac{1}{2}R$$

The **SIGN** convention is to assign a **negative value** to **virtual** quantities.

In Class Demo