

EXAM 3

Name: _____ unid: u _____


Discussion TA (circle): Adam Bill Doug Eric

- A. [6 pts.] A spacecraft has a nonrelativistic (or classical) momentum whose magnitude is $2.70 \times 10^{13} \text{ kg}\cdot\text{m/s}$. The spacecraft moves at such a speed that the pilot measures the proper time interval between two events to be one-eighth the dilated time interval. Find the relativistic momentum of the spacecraft.

$$t' = \gamma t_0 \rightarrow \gamma = 8 \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p_{\text{rel}} = \gamma mv = (8)(2.7 \text{ E } 13 \text{ kg}\cdot\text{m/s}) = 21.6 \text{ E } 13 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

- B. [6 pts.] A resistor is connected across the terminals of a 9.0 V battery, which delivers $1.7 \cdot 10^5 \text{ J}$ of energy to the resistor in 5 hours. What is the resistance of the resistor?



$$P = \frac{V^2}{R} = \frac{E_1}{t} \rightarrow$$

$$R = \frac{V^2 t}{E_1} = \frac{(9\text{V})^2 (5 \cdot 60 \cdot 60 \text{ s})}{1.7 \text{ E } 5 \text{ J}} \approx 8.576 \Omega$$

- C. [6 pts.] Suppose one gallon of gasoline produces $1.1 \times 10^8 \text{ J}$ of energy, and this energy is sufficient to operate a particular car for 28 miles. An aspirin tablet has a mass of 265 mg. If the aspirin could be converted completely into thermal energy, how many miles would this car go on a single tablet?

$1.1 \text{ E } 8 \text{ J} \rightarrow 28 \text{ miles}$ $E_{\text{asp}} = m_{\text{asp}} c^2$

$$\text{miles}_{\text{asp}} = \frac{E_{\text{asp}}}{E_{\text{gas}}} d_{\text{gas}} = \frac{m_{\text{asp}} c^2}{1.1 \text{ E } 8 \text{ J}} 28 \text{ miles} \approx 6 \text{ million miles}$$

- D. [7 pts.] A Wookiee spacecraft has a speed of $0.70 c$ with respect to the earth. The Wookiees measure 29.0 h for the time interval between two events on the earth. What value for the time interval would they measure if their ship had a speed of $0.96 c$ with respect to the earth (in hours)?

for $0.7c$ $t'_1 = \gamma_1 t_0$ with $t'_1 = 29 \text{ hours}$ and $\gamma(0.7c) = 1.4$
 for $0.96c$ $t'_2 = \gamma_2 t_0$ $\gamma(0.96c) = 3.57$

gives

$$t'_2 = \frac{\gamma_2}{\gamma_1} t'_1 \approx \frac{3.57}{1.4} 29 \text{ h} \approx 73.95 \text{ hours} \approx 74 \text{ hours}$$

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- A. [9 pts.] A resistor has a resistance R , and a battery has an internal resistance r . When the resistor is connected across the battery, 13% less power is dissipated in R than would be dissipated if the battery had no internal resistance. Find the ratio r/R .

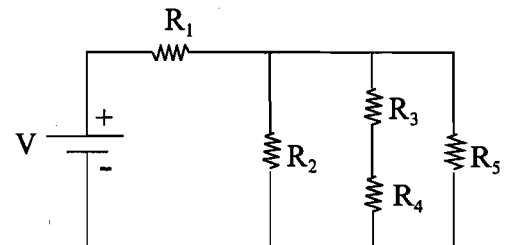
$$P_0 = \frac{\mathcal{E}^2}{R} \text{ (no internal resistance)} \quad P_1 = \frac{V^2}{R} \text{ (with internal resistance)} \quad P_1 = 0.87 P_0$$

$$\frac{V^2}{R} = 0.87 \frac{\mathcal{E}^2}{R} \Rightarrow V = \sqrt{0.87} \mathcal{E} \quad \text{But } V = IR \text{ and } \mathcal{E} = V + Ir$$

$$\text{So } \sqrt{0.87} \mathcal{E} = IR \Rightarrow \mathcal{E} = \frac{IR}{\sqrt{0.87}} \quad \mathcal{E} = V + IR \Rightarrow \frac{IR}{\sqrt{0.87}} = IR + Ir$$

$$R \left(\frac{1}{\sqrt{0.87}} - 1 \right) = r \quad \frac{r}{R} = \left(\frac{1}{\sqrt{0.87}} - 1 \right) = 7.211 \times 10^{-2} \text{ or } 0.07211$$

- B. [7 pts.] The circuit in the drawing contains five identical resistors of Resistance R . The $V = 60 \text{ V}$ battery delivers 87 W of power to the circuit. What is the resistance R of each resistor?



$$P = \frac{V^2}{R_{eq}} \Rightarrow R_{eq} = \frac{V^2}{P}$$

$$R_{3,4} = R + R = 2R$$

$$\frac{1}{R_{2,3,4,5}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{R} \Rightarrow R_{2,3,4,5} = \frac{2R}{5}$$

$$R_{1,2,3,4,5} = R + \frac{2R}{5} = \frac{7R}{5} \quad R_{eq} = \frac{V^2}{P} \Rightarrow \frac{7R}{5} = \frac{(60V)^2}{(87W)} \Rightarrow R = 29.56 \Omega$$

- C. [9 pts.] In a heart pacemaker, a pulse is delivered to the heart 78 times per minute. The capacitor that controls this pulsing rate discharges through a resistance of $1.8 \cdot 10^6 \Omega$. One pulse is delivered every time the fully charged capacitor loses 70.0% of its original charge. What is the capacitance of the capacitor?

$$Q = Q_0 e^{-\frac{t}{RC}} \quad t = \frac{60 \text{ sec}}{78 \text{ times}} = 0.769231 \text{ sec}$$

$$70\% \text{ loss} = 30\% \text{ remains} \quad 0.30 = e^{-\frac{t}{RC}} \Rightarrow \ln(0.30) = -\frac{t}{RC}$$

$$Q = 0.30 Q_0 \quad C = \frac{-t}{R \ln(0.30)} = \frac{-(0.769231 \text{ sec})}{(1.8 \times 10^6 \Omega)(\ln(0.30))} =$$

$$C = 3.5495 \times 10^{-7} \text{ F}$$

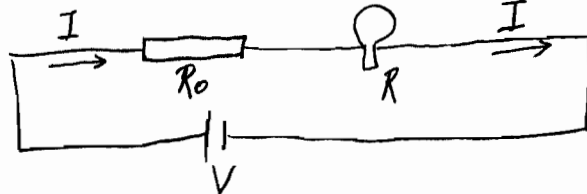
EXAM 3

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- A. [12 pts.] A light bulb is wired in series with a 137Ω resistor, and they are connected across a 120 V source. The power delivered to the light bulb is 22.8 W. What are the two possible resistances of the light bulb?



$$P = I^2 R$$

$$I = \frac{V}{R_0 + R}$$

$$\text{so } P = \left(\frac{V}{R_0 + R} \right)^2 R$$

$$P(R_0^2 + R^2 + 2R_0R) = V^2 R$$

$$PR^2 + (2R_0P - V^2)R + R_0^2P = 0$$

$$P = 22.8 \text{ W}, R_0 = 137 \Omega, V = 120 \text{ V}$$

$$22.8R^2 - 8152.8R + 427933.2 = 0$$

By using $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

one obtains

$$R = 293.67 (\Omega)$$

$$\text{or } R = 63.91 (\Omega)$$

- B. [13 pts.] A portable electric heater uses 18 A of current. The manufacturer recommends that an extension cord attached to the heater dissipate no more than 1.9 W of power per meter of cord length. What is the smallest radius of copper wire that can be used in the extension cord? (Note: An extension cord contains two wires. Resistivity of copper $1.68 \times 10^{-8} \Omega \cdot \text{m}$.)

$$P/L = 1.9 \text{ W/m}$$

$$R = \rho \frac{2L}{\pi r^2}$$

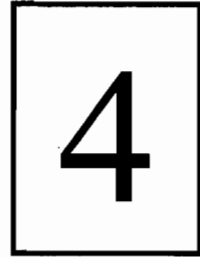
$$P = I^2 R = I^2 \rho \frac{2L}{\pi r^2}$$

$$r = I \sqrt{\frac{2\rho(L)}{\pi(P)}} = 18 \sqrt{\frac{2 \times 1.68 \times 10^{-8}}{\pi} \frac{1}{1.9}}$$

$$= 0.00135 \text{ (m)}$$

$$= 1.35 \times 10^{-3} \text{ (m)}$$

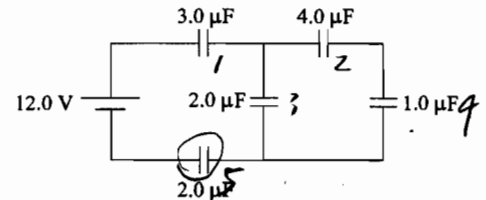
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A. [13 pts.] Given the circuit shown. All capacitors are fully charged.
 How much energy is stored by the $2.0 \mu\text{F}$ capacitor?



$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_0}$$

where $C_0 = C_3 + C'$ and $C' = \frac{C_2 C_4}{C_2 + C_4}$

so $C' = \frac{1 \times 4}{1+4} = 0.8 \mu\text{F}$ and $C_0 = 2 + 0.8 = 2.8 \mu\text{F}$

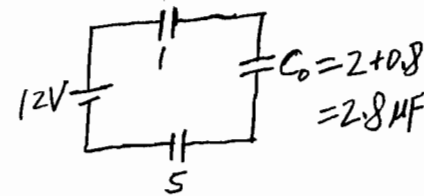
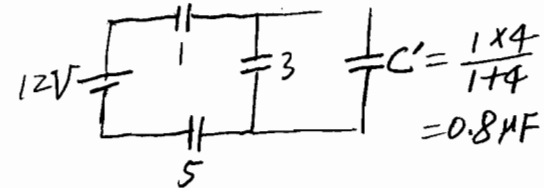
$$C_{tot} = 1 / \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{2.8} \right) = 0.84 \mu\text{F}$$

$$Q = C_{tot} V = 0.84 \times 12 = 10.08 \mu\text{C}$$

$$E_5 = \frac{1}{2} \frac{Q_5^2}{C_5}, \quad Q_5 = Q$$

$$= \frac{1}{2} (10.08)^2 / 2.0$$

$= 25.40 \mu\text{J}$



B. [12 pts.] Examine the circuit shown. What is the power dissipated by the 12Ω resistor?

$$R_{tot} = R_1 + R_0, \quad \frac{1}{R_0} = \frac{1}{R_2 + R_3} + \frac{1}{R_4 + R_5}, \quad R_2 = 3 \Omega, R_3 = 6 \Omega$$

$$R_4 = 6 \Omega, R_5 = 12 \Omega$$

$$R_{tot} = R_1 + \frac{(R_2 + R_3)(R_4 + R_5)}{R_2 + R_3 + R_4 + R_5} = 4 + \frac{9 \times 18}{9 + 18} = 10 \Omega$$

$$I = \frac{V}{R_{tot}} = 120 / 10 = 12 \text{ (A)} \quad V_0 = I R_0 = 12 \times 6 = 72 \text{ V}$$

$$V_0 = V_4 + V_5, \quad \frac{V_4}{V_5} = \frac{R_4}{R_5}$$

$$= V_5 \left(1 + \frac{R_4}{R_5} \right)$$

$$V_5 = V_0 / \left(1 + \frac{R_4}{R_5} \right)$$

$$= 72 / \left(1 + \frac{6}{12} \right) = 48 \text{ (V)}$$

$$P = V_5^2 / R_5$$

$$= 48^2 / 12 = \text{192 (W)}$$

