Physics 2210
Fall 2015

PHYS 2210
Units 21-24 Review
12/11/2015
1. (gi11-015) A 0.25-kg mass at the end of a spring oscillates 2.2 times per second with an amplitude of 0.15 m. Find (a) the equation \( x(\)t\) as a function of \( t \) the\) that describes the motion of the\) mass, assuming that at \( t = 0, x \) was a maximum. (b) the speed when it passes the equilibrium point, (b) the speed when it is 0.10 m from equilibrium, and (d) the total energy of the system.

Answer (a) \( x(t) = (0.15 \text{ m}) \cos(4.4\pi t) \). (b) 2.1 m/s; (c) 1.5 m/s; (d) 0.54 J;
(a) Given: \( m = 0.25 \text{ kg} \), \( f = 2.2 \text{ Hz} \), \( A = 0.15 \text{ m} \)

At \( t = 0 \), \( x = A \) (and \( v = 0 \) is implied: \( \frac{dx}{dt} = 0 \) at max)

General form: \( x = A \cos(\omega t + \phi) \)

\[ x(t = 0) = A \cos \phi = A \rightarrow \phi = 0 \text{ is one answer} \]

\[ x(t) = (0.15 \text{ m}) \cdot \cos \left( \frac{4.4 \pi \text{ s}^{-1} \cdot t}{\text{s}} \right) \]

\( \omega = 2\pi f = 2\pi \left( 2.2 \text{ s}^{-1} \right) = 4.4\pi \text{ s}^{-1} \)

(b) \( v = ? \) when \( x = 0 \)

\[ x = 0 \Rightarrow A \cos \omega t = 0 \Rightarrow \omega t = \frac{\pi}{2} \]

\[ v = \frac{dx}{dt} = -A \omega \sin \omega t = -A \omega \sin \left( \frac{\pi}{2} \right) \]

\[ \Rightarrow \text{speed} = |v| = |\omega A| = (4.4\pi \text{ s}^{-1})(0.15\text{ m}) = 2.07 \text{ m/s} \]

(c) \( x = 0.10 \text{ m} \Rightarrow 0.10 \text{ m} = 0.15 \text{ m} \cdot \cos \omega t \)

\[ \Rightarrow \cos \omega t = \frac{2}{3} \]

Can use \( \sin^2 \omega t = 1 - \cos^2 \omega t = 1 - \frac{4}{9} = \frac{5}{9} \)

\[ \Rightarrow \sin \omega t = 0.7454 \]

\[ \Rightarrow |v| = |\omega A \sin \omega t| = (4.4\pi \text{ s}^{-1})(0.15\text{ m})(0.7454) = 1.55 \text{ m/s} \]

(d) Total energy = \( U \) when \( x = A \) (and \( v = 0 \)).

\[ U_{\text{max}} = \frac{1}{2} k (x_{\text{max}})^2 = \frac{1}{2} k A^2 \]

\[ v_{\text{max}} = \frac{1}{2} (m v_{\text{max}}^2) A^2 \]

\[ U_{\text{max}} = \frac{1}{2} (0.25 \text{ kg})(4.4\pi \text{ s}^{-1})^2 (0.15 \text{ m})^2 \]

\[ E = 0.537 \text{ J} \]
2. (hr15-033) A block of mass $M = 5.4$ kg, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of constant $k = 6000$ N/m. A bullet of mass $m = 9.5$ g and velocity $\vec{v}$ of magnitude 630 m/s strikes and is embedded in the block (see figure).
Assuming the compression of the spring is negligible until the bullet is embedded, determine (a) the speed of the block immediately after the collision and (b) the amplitude of the resulting simple harmonic motion.

Answer: (a) 1.1 m/s; (b) 3.3 cm
(a) Totally inelastic collision
(2 objects stick after collision)
Total momentum is conserved
\[ P_i = mV_i = (9.5 \times 10^{-3} \text{ kg}) (6 \times 10^3 \text{ m/s}) = 5.795 \text{ kg m/s} \]
\[ P_f = (m+M) V_f = (5.4095 \text{ kg}) V_f \]
\[ P_i = P_f \Rightarrow V_f = \frac{5.795 \text{ kg m/s}}{5.4095 \text{ kg}} = 1.068 \text{ m/s} \]

(b) \( x = A \cos(\omega t + \phi) \)
data at \( t = 0 \) (collinear) \( x = 0 \) \( V = V_f = 1.106 \text{ m/s} \)
but \( x(0) = A \cos \phi \Rightarrow \cos \phi = 0 \)
\[ V = \frac{dx}{dt} = -wA \sin(\omega t + \phi) \]
\[ V(0) = -wA \sin \phi = +1.106 \text{ m/s} \]
\[ \Rightarrow \sin \phi = -1 \Rightarrow wA = V_f = 1.106 \text{ m/s} \]
\[ w = \sqrt{\frac{k}{M_{tot}}} = \sqrt{\frac{6030 \text{ N/m}}{5.4095 \text{ kg}}} = 33.3 \text{ s}^{-1} \]
\[ A = \frac{1.106 \text{ m/s}}{33.3 \text{ s}^{-1}} = \boxed{0.0332 \text{ m}} = 3.32 \text{ cm} \]

Alternately: energy is conserved
\[ K_i = \frac{1}{2} m V_i^2 \]
\[ U_i = 0 \]
\[ \Rightarrow \frac{1}{2} m V_i^2 = \frac{1}{2} kA^2 \]
\[ \Rightarrow A^2 = \frac{m V_i^2}{k} = \frac{5.4095 \text{ kg}}{6030 \text{ N/m}} \cdot (1.106 \text{ m/s})^2 \]
\[ A = 0.0332 \text{ m} = 3.32 \text{ cm} \]
6. (hr016-023) A sinusoidal transverse wave is traveling along a string in the negative direction of an x axis. The figure shows a snapshot of the displacement as a function of position at time $t = 0$; the scale of the $y$ axis is set by $y_s = 4.0$ cm. The string tension is 3.6 N, and its linear density is 25 g/m. Find the (a) amplitude, (b) wavelength, (c) wave speed, and (d) period of the wave. (e) Find the maximum transverse speed of a particle in the string. If the wave is of the form 

$$y(x, t) = y_m \sin(kx \pm \omega t + \varphi),$$

what are (f) $k$, (g) $\omega$, (h) $\varphi$, and (i) the correct choice of sign in front of $\omega$?

NOTE this problem uses “SIN” instead of “COS”

Answer: (a) 5.0 cm; (b) 40 cm; (c) 12 m/s; (d) 0.033 s; (e) 9.4 m/s; (f) 16 m$^{-1}$; (g) $1.9 \times 10^2$ s$^{-1}$; (h) 0.93 rad; (i) plus
(6) Figure at \( t = 0 \), \( T = 3.6 \text{N} \), \( M = 0.025 \text{kg} \)

\[ \lambda = 4.0 \text{cm} = 0.04 \text{m} = 4 \text{ ticks} \Rightarrow 1 \text{ tick} = 0.01 \text{ m} \]

(a) \( A = 5 \text{ ticks} = 0.05 \text{ m} = 5 \text{ cm} \)

(b) From graph: coast \( \rightarrow \text{coast} = 4 \text{ horizontal ticks} \)

\[ \lambda = 40 \text{ cm} = 0.40 \text{ m} \]

(c) \( v = \sqrt{\frac{T}{M}} = \sqrt{\frac{3.6 \text{ N}}{0.025 \text{ kg}}} = 12.0 \text{ m/s} \)

(d) \( T = \frac{1}{f} \) but \( v = f \lambda \Rightarrow f = \frac{v}{\lambda} \)

\[ \Rightarrow T = \frac{1}{f} = \frac{\lambda}{v} = \frac{0.40 \text{ m}}{12 \text{ m/s}} = 0.0333 \text{s} \]

(e) transverse speed: \( v = \frac{dy}{dt} = \frac{dY}{dt} \)

\[ Y = Y_m \sin (kx + \omega t + \phi) \Rightarrow \frac{dY}{dt} = \pm \omega Y_m \cos (kx + \omega t + \phi) \]

\[ v_{y\text{max}} = \omega Y_m = \frac{2\pi}{T} \cdot Y_m = \frac{2\pi}{T} \cdot A \]

\[ v_{y\text{max}} = \frac{2\pi}{0.0333 \text{s}} \cdot 0.05 \text{ m} = 1.425 \text{ m/s} \]

(f) \( k = \frac{2\pi}{\lambda} = 15.7 \text{ m}^{-1} = \frac{2\pi}{0.40 \text{ m}} \)

(g) \( \omega = \frac{2\pi}{T} = \frac{2\pi}{0.0333 \text{s}} = 188.5 \text{ s}^{-1} \)

(i): Wave \( \leftarrow (-x \sin t) \)

\[ \Rightarrow Y = Y_m (kx + \omega t + \phi) \]

Thus!
(2) \[ Y = Y_m \sin (kx + \omega t + \phi) \]

at \( t = 0 \)

\[ Y = Y_m \sin (kx + \phi) \]
plot is a sine wave shifted to left

\[ \Rightarrow \phi > 0 \]

at \( x = 0 \)

\[ Y = Y_m \sin \phi \]
\[ Y = 0.04 \text{ m} \]

\[ \Rightarrow \frac{0.04 \text{ m}}{0.105 \text{ m}} = \sin \phi = 0.38 \]

\[ \phi = 0.1927 \text{ rad} \]
5. (hr016-009) A sinusoidal wave moving along a string is shown twice in the figure, as crest A travels in the positive direction of an x axis by distance \( d = 6.0 \) cm in \( 4.0 \) ms (milliseconds). The tick marks along the axis are separated by 10 cm; height \( H = 6.00 \) mm.

The equation for the wave is in the form \( y(x, t) = y_m \sin(kx \pm \omega t) \), so what are (a) \( y_m \), (b) \( k \), (c) \( \omega \), and (d) the correct choice of sign in front of \( \omega \)?

**NOTE** this problem uses “SIN” instead of “COS”

**Answer:** (a) 3.0 mm; (b) 16 m\(^{-1}\); (c) \( 2.4 \times 10^2 \) s\(^{-1}\); (d) minus
(5) given: wave travel, 6.0 cm \( \Rightarrow \) wave speed = \( V = \frac{6.0 \times 10^{-2} \text{ m}}{4.0 \times 10^{-3} \text{ s}} = 15 \text{ m/s} \)

(a) \( Y_m = \frac{1}{2} H \) as shown
\[ Y_m = \frac{6.0 \text{ mm}}{2} = 3.0 \text{ mm} = \frac{3.0 \times 10^{-3} \text{ m}}{} \]

(b) \( k = \frac{2\pi}{\lambda} \)

From graph: \( \lambda = 4 \times \text{tick spacing} \)
\[ = 4 \times 10 \text{ cm} = 40 \text{ cm} = 0.40 \text{ m} \]
\[ k = \frac{2\pi}{0.40 \text{ m}} = 5\pi \text{ m}^{-1} = 15.7 \text{ m}^{-1} \]

(c) \( \omega = 2\pi f \), \( V = f \lambda = \left( \frac{\omega}{2\pi} \right) \cdot \left( \frac{2\pi}{k} \right) = \frac{\omega}{k} \)
\[ \Rightarrow \omega = V k \]
\[ = (15 \text{ m/s}) (5\pi \text{ m}^{-1}) = 236 \text{ s}^{-1} \]

(d) \( \text{travels } (+x \ \text{ dir}) \ \Rightarrow \ Y = Y_m \sin(kx - \omega t) \pm \text{ MINUS!} \)