IE Spring Loaded collision

Wording

A cart with mass $m_1 = 3.2\ kg$ and initial velocity of $v_{1,i} = 2.1\ m/s$ collides with another cart of mass $M_2 = 4.3\ kg$ which is initially at rest in the lab frame. The collision is completely elastic, and the wheels on the carts can be treated as massless and frictionless. What is the velocity of $m_1$ in the center of mass frame after the collision? See Figure 1.

Solution

Center of mass is moving with

$$v_{cm} = \frac{m_1 v_1 + M_2 v_2}{m_1 + M_2}$$

The velocity of $m_1$ before the collision in the center of mass frame is thus

$$v_{1,i} = v_{1,i} - v_{cm}$$

Since the collision is elastic, the velocity will reverse in the center of mass frame, i.e.

$$v_f^* = -v_i^*$$

$$v_f^* = -1.204\ m/s$$
Bumper Cars

Wording

Figure 2: Bumper Cars

A bumper car with mass \( m_1 = 103 \text{ kg} \) is moving to the right with a velocity of \( v_1 = 4 \text{ m/s} \). A second bumper car with mass \( m_2 = 92 \text{ kg} \) is moving to the left with a velocity of \( v_2 = -3.4 \text{ m/s} \). The two cars have an elastic collision. Assume the surface is frictionless. See Figure 2.

1. What is the velocity of the center of mass of the system?
2. What is the initial velocity of car 1 in the center-of-mass reference frame?
3. What is the final velocity of car 1 in the center-of-mass reference frame?
4. What is the final velocity of car 1 in the ground (original) reference frame?
5. What is the final velocity of car 2 in the ground (original) reference frame?
6. In a new (inelastic) collision, the same two bumper cars with the same initial velocities now latch together as they collide. What is the final speed of the two bumper cars after the collision?
7. Compare the loss in energy in the two collisions.

Solution

1)

Center of mass velocity is

\[
v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}
\]

\[
v_{cm} = 0.509 \text{ m/s}
\]

2)

Initial velocity of the car 1 in the center-of-mass frame

\[
v_{1,i}^{(cm)} = v_1 - v_{cm}
\]

\[
v_{1,i}^{(cm)} = 3.491 \text{ m/s}
\]
3) Since the collision is elastic, the car 1 will bounce off with opposite velocity, i.e.

$$v_{1,f}^{(cm)} = -3.491 \text{ m/s}$$

4) In the ground frame, we have

$$v_{1,f}^{(G)} = v_{cm} + v_{1,i}^{(cm)}$$

$$v_{1,f}^{(G)} = -2.983 \text{ m/s}$$

5) The momentum is conserved, i.e.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Thus

$$v_2' = \frac{m_1}{m_2} (v_1 - v_1') + v_2$$

where the values are taken in the ground frame.

$$v_{2,f}^{(G)} = 4.417 \text{ m/s}$$

6) Momentum is still conserved

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_{12}$$

So we get

$$v_{12} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} m_1 + m_2$$

$$v_{12,f} = 0.509 \text{ m/s}$$

7) In the first case, there is no energy loss - the collision is elastic!

In the second case, we have

$$\Delta E = KE_f - KE_i = \frac{1}{2} (m_1 + m_2) v_{12}^2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2$$
Tipler6 8.P.061.

Wording

A proton of mass \( m \) is moving with initial speed \( v_0 \) directly toward the center of a nucleus of mass \( 9m \), which is initially at rest. Because both carry positive electrical charge, they repel each other. Find the speed of the nucleus for the following conditions.

1. when the distance between the two is at its smallest value
2. when the distance between the two is large

Solution

(a) Conservation of momentum gives

\[
p_p + p_n = p'_p + p'_n
\]

When the distance is the smallest, proton and nucleus are moving with the same velocity \( v' := v'_p = v'_n \)

\[
mv_0 + 0 = mv' + 9mv'
\]

which yields

\[
v' = 0.1 \times v_0
\]

(b)

Here again, we have conservation of momentum

\[
mv_0 = p_p + p_n = p''_p + p''_n = mv''_p + 9mv''_n
\]

which yields

\[
v''_p = v_0 - 9v''_n
\]

But this time, also conservation of energy applies

\[
\frac{1}{2}mv_0^2 = KE_p + KE_n = KE''_p + KE''_n = \frac{1}{2}m(v''_p)^2 + \frac{1}{2}(9m)(v''_n)^2
\]

and gives

\[
v_0^2 = (v''_p)^2 + 9(v''_n)^2
\]

Combining the two results

\[
0 = (5v''_n - v_0)v''_n
\]

that gives non-zero solution

\[
v''_n = 0.2 \times v_0
\]

1 Zero solution is basically the starting velocity.
Tipler6 8.P.062.

Wording
An electron collides elastically with a hydrogen atom that is initially at rest. Assume all the motion occurs along a straight line. What fraction of the electron’s initial kinetic energy is transferred to the atom? (Take the mass of the hydrogen atom to be 1840 times the mass of an electron.)

Solution
Conservation of momentum
\[ mv_e = mv_e' + 1840mv_H \implies v_H = \frac{v_e - v_e'}{1840} \]

Conservation of energy
\[ \frac{1}{2}mv_e^2 = \frac{1}{2}mv_e'^2 + \frac{1}{2}(1840m)v_H^2 \]

If we divide by the initial kinetic energy of the electron, we get
\[ 1 = \left( \frac{v_e'}{v_e} \right)^2 + \eta \]
where \( \eta \) is the fraction. Also
\[ v_e^2 = v_e'^2 + 1840v_H^2 \]

Combining previous equations, we obtain
\[ \left( \frac{v_e'}{v_e} \right)^2 - \left( \frac{v_e'}{v_e} \right) - \frac{1839}{2} = 0 \]

Solution will be put into
\[ \eta = 1 - \left( \frac{v_e'}{v_e} \right)^2 \]
\[ \eta = 0.217\% \]

Tipler6 8.P.103.

Wording
A steel ball of mass \( m_1 = 0.9\, kg \) and a cord of length of \( L = 2\, m \) of negligible mass make up a simple pendulum that can pivot without friction about the point O, as in the figure below. This pendulum is released from rest in a horizontal position, and when the ball is at its lowest point it strikes a block of mass \( m_2 = 0.9\, kg \) sitting at rest on a shelf. Assume that the collision is perfectly elastic and that the coefficient of kinetic friction between the block and shelf is 0.10. See Figure 3.

1. What is the velocity of the block just after impact?

2. How far does the block slide before coming to rest (assuming that the shelf is long enough)?
Solution

(a)
Velocity right before the impact is determined by conservation of the energy

\[ m_1 g L = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gL} \]

During the collision, the momentum is conserved, i.e.

\[ m_1 v_1 = m_1 v_1' + m_2 v_2' \]

as is the energy

\[ \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \]

Eliminating \( v_1' \) gives

\[ v_2' = \frac{2m_1}{m_1 + m_2} \sqrt{2gL} \]

\[ v_2' = 6.264 \, \text{m/s} \]

(b)
Since this is a motion with a constant acceleration, we can use

\[ v_f^2 - v_i^2 = 2aD \]

where \( v_f = 0 \), \( v_i = v_2' \). Acceleration is given by friction

\[ m_2 a = -\mu_k N = -\mu_k m_2 g \]

which yields

\[ D = \frac{v_f^2}{2\mu_k g} \]

\[ D = 20 \, \text{m} \]
Billiard Balls

Wording

A white billiard ball with mass $m_w = 1.32\, \text{kg}$ is moving directly to the right with a speed of $v = 2.9\, \text{m/s}$ and collides elastically with a black billiard ball with the same mass $m_b = 1.32\, \text{kg}$ that is initially at rest. The two collide elastically and the white ball ends up moving at an angle above the horizontal of $\theta_w = 69^\circ$ and the black ball ends up moving at an angle below the horizontal of $\theta_b = 21^\circ$. See Figure 4.

1. What is the final speed of the white ball?
2. What is the final speed of the black ball?
3. What is the magnitude of the final total momentum of the system?
4. What is the final total energy of the system?

Solution

The momentum is conserved

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

So in components

$$m_w v = m_w v'_w \cos \theta_w + m_b v'_b \cos \theta_b$$
$$0 = m_w v'_w \sin \theta_w - m_b v'_b \sin \theta_b$$

And so is the energy

$$\frac{1}{2} m_w v_w^2 = \frac{1}{2} m_w v'_w^2 + \frac{1}{2} m_b v'_b^2$$

Solving for speeds gives

1) $v'_w = 1.039\, \text{m/s}$
2) \[ v'_b = 2.707 \text{ m/s} \]

3) Total momentum of the system is conserved
\[ P = mv_w \]
\[ P = 3.828 \text{ kg} \cdot \text{m/s} \]

4) Total energy is conserved
\[ E = \frac{1}{2} m_w v_w^2 \]
\[ E = 5.5506 \text{ J} \]