Physics 2210
Fall 2015

smartPhysics

11 Conservation of Momentum
12 Elastic Collisions

10/23/2015
Ben je er klaar voor?
Daar gaan we dan.

https://www.youtube.com/watch?v=uWF20u3FUo4
A more controlled set of experiments

https://www.youtube.com/watch?v=Zy7JQ0lwj1w
Conservation of Momentum in an Explosion

Explosion Experiment: A cart of mass $m_1$ is tied to a second cart of mass $m_2$. They are initially at rest, and an explosive device (spring) is released between them and they move apart at speeds $v_1 (-x \text{ dir.})$ and $v_2 (+x \text{ dir.})$. Find the ratio $v_2 / v_1$.

Case 2:

$m_2 = \frac{1}{2} m_1$

$m_2 = 1.0 \text{ SC}$

$m_1 = 2.0 \text{ SC}$

From http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chptr3_energy.htm
Cart on left has extra mass on it: making it twice that of the one on the right.
Explosion: $m_2 = 0.5 \, m_1$

Digitized $x_1$ and $x_2$ every 3 frames (0.10 s)
Explosion: \( m_2 = 0.5 \ m_1 \)

<table>
<thead>
<tr>
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<th>x2 (pix)</th>
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<tr>
<td>0.7</td>
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</tbody>
</table>

Here \( m_1 = 2.0 \ SC \) (Standard Cart), \( m_1 = 1.0 \ SC \)

Then Before the explosion:

\[
\vec{p}_{1i} = (2.0 \ SCU)*(0.0 \ pix/s) \ \hat{i} = 0.0 \ SC \cdot \text{pix/s} \ \hat{i}, \quad \text{and}
\]

\[
\vec{p}_{2i} = (1.0 \ SCU)*(0.0 \ pix/s) \ \hat{i} = 0.0 \ SC \cdot \text{pix/s} \ \hat{i}
\]

So the initial total momentum is

\[
\vec{P}_i = \vec{p}_{1i} + \vec{p}_{2i} = 0.0 \ SC \cdot \text{pix/s} \ \hat{i}
\]

zero !!!

After the explosion:

\[
\vec{p}_{1f} = (2.0 \ SC)*(-227 \ \text{pix/s}) \ \hat{i}
\]

\[
= -454 \ SCU \cdot \text{pix/s} \ \hat{i}, \quad \text{and}
\]

\[
\vec{p}_{2f} = (1.0 \ SC)*(+444 \ \text{pix/s}) \ \hat{i}
\]

\[
= +444 \ SCU \cdot \text{pix/s} \ \hat{i}
\]

So the initial total momentum is

\[
\vec{P}_f = \vec{p}_{1f} + \vec{p}_{2f} = -10 \ SC \cdot \text{pix/s} \ \hat{i}
\]

Within a few % of \( p_{1f} \) and \( p_{2f} \) of being zero !!!
Explosion Case 2: \( m_2 = 0.5 \, m_1 \)

Digitized \( x_1 \) and \( x_2 \) every 3 frames (0.10 s)

Since \( m_2 = 0.5 \, m_1 \) then

\[
X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + \frac{1}{2} m_1 x_2}{m_1 + \frac{1}{2} m_1}
\]

\[
= \frac{x_1 + 0.5 x_2}{1.5}
\]

Note the center-of-mass of the system remained stationary

Measurement of V1, V2:
(from slope near \( t=0 \))

\( V1 = 227 \) pix/s
\( V2 = 444 \) pix/s

\( V2/V1 = 1.96 \)

Within few % of prediction of 2 !!!
Elastic Collisions

If the only forces acting during a collision are conservative forces, then the kinetic energy of the system, defined to be the sum of the kinetic energies of the colliding objects, is conserved. Such collisions are called elastic collisions.

\[
\sum K_i = \sum K_f
\]
Center-of-Mass Frame

Elastic collisions are most simply described in the center-of-mass reference frame of the colliding objects.

The collision may cause objects to be deflected through some angle in the frame, but their speeds will always remain the same.
A box sliding on a frictionless surface collides and sticks to a second identical box which is initially at rest.

Compare the initial and final kinetic energies of the system of two boxes.

A. $K_{\text{initial}} > K_{\text{final}}$
B. $K_{\text{initial}} = K_{\text{final}}$
C. $K_{\text{initial}} < K_{\text{final}}$
Conservation of Momentum in a Collision

Collision Experiment: A cart of mass $m_1$ is traveling at speed $v_i$ in the $+x$ direction towards a second cart of mass $m_2$, which is at rest. They collide and stick together. What is their (common) speed $v_f$ after the collision?

Case 1:

$m_2 = m_1 = 1.0$ SC

This is called a “totally inelastic collision”

From http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chp3r3_energy.htm
Conservation of Momentum in a Collision

Theory: Total momentum is conserved in the $x$ direction because no external forces with non-zero $x$-components act on the group (they interact but the internal forces must cancel because of N3L)

\[ P_{ix} = m_1 v_i \]
\[ P_{fx} = (m_1 + m_2) v_f \]

Setting $P_{ix} = P_{fx}$ we get

\[ m_1 v_i = (m_1 + m_2) v_f \]

So the final speed of the conjoined carts is given by

\[ v_f = \frac{m_1}{m_1 + m_2} v_i \]

Or: the ratio $v_f/v_i$ is given by

\[ \frac{v_f}{v_i} = \frac{m_1}{m_1 + m_2} \]

Predictions for three cases

(1) $m_1 = 1.0$ SC, $m_2 = 1.0$ SC: $v_f/v_i = 1/2 = 0.500$

(2) $m_1 = 2.0$ SC, $m_2 = 1.0$ SC: $v_f/v_i = 2/3 = 0.667$

(3) $m_1 = 1.0$ SC, $m_2 = 2.0$ SC: $v_f/v_i = 1/3 = 0.333$
Case 1: $m_1 = m_2$

This window dump missed the first digitized point.
Totally Inelastic Collision: \( m_2 = m_1 \)

We have \( m_1 = 1.0 \) SC \( m_2 = 1.0 \) SC

Before the collision:
\[
\vec{p}_{1i} = (1.0 \text{ SCU}) \times (342 \text{ pix/s}) \hat{i} \\
= 342 \text{ SC} \cdot \text{pix/s} \hat{i} \\
\vec{p}_{2i} = (1.0 \text{ SCU}) \times (0.0 \text{ pix/s}) \text{SC} \cdot \text{pix/s} \hat{i} \\
= 0 \hat{i} \\
\vec{P}_i = \vec{p}_{1i} + \vec{p}_{2i} = 342 \text{ SC} \cdot \text{pix/s} \hat{i}
\]

After the collision:
\[
\vec{P}_f = (1.0 + 1.0) \text{ SCU} \times (168 \text{ pix/s}) \hat{i} \\
= 336 \text{ SC} \cdot \text{pix/s} \hat{i}
\]

Predicted \( \frac{v_f}{v_i} = 1/2 = 0.500 \)

Measured: \( \frac{v_f}{v_i} = 168/342 \)

\( = 0.491. \)

Within a few % of \( P_i \) !!!

Within a few % of prediction !!!
**Totally Inelastic Collision: m2 = m1**

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\[
K_i = \frac{1}{2}(1.0 \text{ SC})(342 \text{ pix/s})^2 + \frac{1}{2}(1.0 \text{ SC})(0 \text{ pix/s})^2
= 5.85 \times 10^4 \text{ SC}\cdot\text{pix/s}
\]

\[
K_i = \frac{1}{2}(2.0 \text{ SC})(168 \text{ pix/s})^2
= 2.82 \times 10^4 \text{ SC}\cdot\text{pix/s}
\]

**Lost ~50% of kinetic energy**
**Totally Inelastic Collision:  \( m_2 = m_1 \)**

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<tr>
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</table>

**Before Collision:**

\[ x_2 = \text{constant} = 386 \ \text{pix} \]

Since \( m_2 = m_1 \) then

\[
X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_1 x_2}{m_1 + m_1} = \frac{x_1 + x_2}{2}
\]

**After the collision the stuck-together body is moving at a final velocity \( V_f \) equal to the (initial) \( V_{CM} \)**
A green block of mass $m$ slides to the right on a frictionless floor and collides elastically with a red block of mass $M$ which is initially at rest. After the collision the green block is at rest and the red block is moving to the right.

How does $M$ compare to $m$?

A. $m > M$
B. $m = M$
C. $m < M$
Conservation of Momentum in Elastic(?) Collisions

Collision Experiment: A cart of mass $m_1$ is traveling with velocity $+v_i$ (in the $+x$ direction) towards a second cart of mass $m_2$, which is at rest. They collide **elastically**. What are their velocities after the collision?

Case 1

$m_1 = 1.0 \text{ SC}$
$m_2 = 1.0 \text{ SC}$

From http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chptr3_energy.htm
(Almost) Elastic Collision Case 1: \( m_2 = m_1 \)

Measurement of \( V_1, V_2 \): (from slope near time of collision)

\[
V_{1i} = 425 \text{pix/s} \quad V_{1f} = 27 \text{ pix/s} \quad V_{2f} = 378 \text{ pix/s}
\]

\[
P_i = (1.0 \text{ SC})(425 \text{ pix/s}) + (1.0 \text{SC})(0 \text{ pix/s}) = 425 \text{ SC} \cdot \text{pix/s}
\]

\[
P_f = (1.0 \text{ SC})(27 \text{ pix/s}) + (1.0 \text{SC})(378 \text{ pix/s}) = 405 \text{ SC} \cdot \text{pix/s}
\]

\( P_f \) within few \% of \( P_i \)

Normally momentum has units of \( \text{kg} \cdot \text{m/s} \)

\[
K_i = (1/2)(1.0 \text{ SC})(425 \text{ pix/s})^2 + (1/2)(1.0 \text{ SC})(0 \text{ pix/s})^2
\]

\[= 9.03 \times 10^4 \text{ SC} \cdot \text{pix/s} \]

\[K_f = (1/2)(1.0 \text{ SC})(27 \text{ pix/s})^2 + (1/2)(1.0 \text{ SC})(378 \text{ pix/s})^2
\]

\[= 7.18 \times 10^4 \text{ SC} \cdot \text{pix/s} \]

Lost \~20\% of kinetic energy
(Almost) Elastic Collision Case 1: $m_2 = m_1$

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Digitized $x_1$ and $x_2$ every 3 frames (0.10 s)

Since $m_2 = 2$ (SC) and $0.5 m_1 = 1.0$ (SC) then

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2x_1 + x_2}{3}$$

The center-of-mass remained at constant velocity through the collision
Conservation of Momentum in Elastic(?) Collisions

Collision Experiment: A cart of mass $m_1$ is traveling with velocity $+v_i$ (in the $+x$ direction) towards a second cart of mass $m_2$, which is at rest. They collide elastically. What are their velocities after the collision?

Case 2
$m_1 = 2.0$ SC
$m_2 = 1.0$ SC

http://www.physics.utah.edu/~jui/2210_s2015/collision02/elastic_cars_bs_x264.avi

From http://physics.wfu.edu/demolabs/demos/avimov/bychptr/chptr3_energy.htm
(Almost) Elastic Collision Case 2: $m_1 = 2m_2$

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Measurement of V1, V2: (from slope near time of collision)

$V_{1i} = V_i = 340 \text{ pix/s}$ \hspace{1cm} $V_{1f} = 133 \text{ pix/s}$ \hspace{1cm} $V_{2f} = 421 \text{ pix/s}$

$p_i = (2.0 \text{ SC})(340 \text{ pix/s}) + (1.0 \text{ SC})(0 \text{ pix/s}) = 680 \text{ SC} \cdot \text{pix/s}$

$p_f = (2.0 \text{ SC})(133 \text{ pix/s}) + (1.0 \text{ SC})(421 \text{ pix/s}) = 687 \text{ SC} \cdot \text{pix/s}$

$p_f$ Within few % of $p_i$

Normally momentum has units of kg·m/s

$K_i = \frac{1}{2}(2.0 \text{ SC})(340 \text{ pix/s})^2 + \frac{1}{2}(1.0 \text{ SC})(0 \text{ pix/s})^2$

$= 1.16 \times 10^5 \text{ SC} \cdot (\text{pix/s})^2$

$K_f = \frac{1}{2}(2.0 \text{ SC})(133 \text{ pix/s})^2 + \frac{1}{2}(1.0 \text{ SC})(421 \text{ pix/s})^2$

$= 1.06 \times 10^5 \text{ SC} \cdot (\text{pix/s})^2$

Lost ~10% of kinetic energy
(Almost) Elastic Collision Case 2: $m_1 = 2m_2$

Digitized $x_1$ and $x_2$ every 3 frames (0.10 s)

Since $m_2 = 2$ (SC) and $0.5m_1 = 1.0$ (SC)

then

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{2x_1 + x_2}{3}$$

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<td>444</td>
<td>673</td>
<td>522.97</td>
</tr>
</tbody>
</table>

The center-of-mass remained at constant velocity through the collision
The collision as seen in the center of mass frame.
Two-Body Collisions in Center-of-Mass Frame

- Figure shows a totally inelastic collision of two particles (mass $m_1, m_2$) with initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$, and common final velocity $\vec{v}_f$.

- $\vec{V}_{CM}$ (constant!) for this system of two particles:

  $$\vec{V}_{CM} = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2}$$

- If a body is moving at velocity $\vec{v}$ then its velocity in the CM frame (i.e. relative to the CM) would be

  $$\vec{v}' = \vec{v} - \vec{V}_{CM}$$

- In the CM frame $\vec{v}'_{1i}$ is given by

  $$\vec{v}'_{1i} = \vec{v}_{1i} - \vec{V}_{CM} = \vec{v}_{1i} - \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2} = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} - m_1 \vec{v}_{1i} - m_2 \vec{v}_{2i}}{m_1 + m_2} = \frac{m_2 \vec{v}_{1i} - m_2 \vec{v}_{2i}}{m_1 + m_2} = \frac{m_2}{m_1 + m_2} (\vec{v}_{1i} - \vec{v}_{2i})$$

- Similarly for $\vec{v}'_{2i}$

  $$\vec{v}'_{2i} = \vec{v}_{2i} - \vec{V}_{CM} = \vec{v}_{2i} - \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2} = \frac{m_1 \vec{v}_{2i} + m_2 \vec{v}_{2i} - m_1 \vec{v}_{1i} - m_2 \vec{v}_{2i}}{m_1 + m_2} = \frac{m_1 \vec{v}_{2i} - m_1 \vec{v}_{1i}}{m_1 + m_2} = \frac{m_1}{m_1 + m_2} (\vec{v}_{2i} - \vec{v}_{1i})$$
Momenta in Center-of-Mass Frame

- In the CM frame the initial velocities are given by
  \[ \vec{v}'_{1i} = \frac{m_2}{m_1 + m_2} (\vec{v}_{1i} - \vec{v}_{2i}), \quad \vec{v}'_{2i} = \frac{m_1}{m_1 + m_2} (\vec{v}_{2i} - \vec{v}_{1i}) \]

- So the corresponding momenta in the CM frame are
  \[ \vec{p}'_{1i} = m_1 \vec{v}'_{1i} = \frac{m_1 m_2}{m_1 + m_2} (\vec{v}_{1i} - \vec{v}_{2i}) \]
  \[ \vec{p}'_{2i} = m_2 \vec{v}'_{2i} = \frac{m_1 m_2}{m_1 + m_2} (\vec{v}_{2i} - \vec{v}_{1i}) \]

- Note that \( \vec{p}'_{1i} = -\vec{p}'_{2i} \), i.e. that \( \vec{p}'_{1i} + \vec{p}'_{2i} = 0 \). The fact that the CM frame momenta of the particles in the group sum to ZERO applies generally to a system of ANY NUMBER of particles (not just 2). For this reason, the CM frame is often known as the zero momentum system (ZMS)

- We often write these CM frame momenta for a system of two particles as
  \[ \vec{p}'_1 = -\mu \vec{v}_{21}, \quad \vec{p}'_2 = +\mu \vec{v}_{21} \]
  where \( \mu = \frac{m_1 m_2}{m_1 + m_2} \) is known as the “reduced mass” (defined only for a system of two particles) and \( \vec{v}_{21} = \vec{v}_2 - \vec{v}_1 \) is the velocity of particle 2 relative to particle 1 (this quantity is independent of the frame of reference)

NOTE the reduced mass + relative velocity description does not apply to systems with more than 2 particles
Totally Inelastic Collision in CM Frame

- In the lab frame, by conservation of momentum
  \[(m_1 + m_2)\vec{v}_f = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}\]
- As expected this means for this totally inelastic collision
  \[\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{(m_1 + m_2)} \equiv \vec{V}_\text{CM}\]
- And so in the CM frame we have
  \[\vec{v}'_f = \vec{v}_f - \vec{V}_\text{CM} = 0\]

Description of the collision by an observer in the CM frame:
1. The two particles travel toward one another with equal and opposite momenta (equal in magnitude but opposite in direction)
2. They collide in a totally inelastic fashion and stick together
3. The resulting composite particle ends up at rest.