SHOW ALL WORK!!
Place answers in box provided for each question. Specify units for each answer.
Report all numbers to two significant figures.
Circle correct answer(s) for each question!

A. [5 pts.] Which of the following are true. Circle all correct answers.

(a) Simple harmonic motion is always due to the oscillatory (back and forth) motion of springs.

(b) Gravitational forces may result in simple harmonic motion.

(c) The angular frequency of a mass m attached to a spring of spring constant k in a container containing a viscous medium is given by \( \omega = \sqrt[k/m]{} \).

(d) A physical pendulum consisting of a large uniform stick of mass m is oscillating with an angular frequency \( \omega \) about a pivot point near one end. If the pivot point is changed to the center of mass, the angular frequency will be unchanged because the moment of inertia, the mass m, and the length of the stick are all unchanged.

(e) None of the above are true.

B. [5 pts.] Which of the following are true. Circle all correct answers.

(a) A figure skater brings in her arms to increase her moment of inertia.

(b) The direction of the torque vector is always perpendicular to the force that created it.

(c) It is possible for the angular momentum to vary with time even if the external torque is zero (and constant).

(d) The reason that we can use conservation of energy for a rolling wheel is that the friction force does not result in a torque about the center of mass.

(e) All of the above are false.

C. [5 pts.] In an isolated system of two bodies that exert gravitational forces on one another and move in elliptical orbits, which of the following remain constant?

(a) The total energy of the system.

(b) The total angular momentum of the system.

(c) The speed of each body.

(d) The gravitational force between the two bodies.

(e) None of the above.
D. \[5 \text{ pts.}\] Calculate the value of \( g \) for a 2.1 kg mass at a height \( 1.32 \times 10^4 \) km above the earth’s surface.

\[
mg = \frac{GM_E}{(R_E + h)^2} \Rightarrow g = \frac{GM_E}{(R_E + h)^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(1.637 + 1.32) \times 10^{10}} = 1.04 \text{ m/s}^2
\]

\boxed{1.04 \text{ m/s}^2}

E. \[5 \text{ pts.}\] A spring of spring constant \( k \) is attached to a fixed point \( P \) on a smooth inclined plane of angle \( \theta = 30^\circ \). When a mass \( m = 2.4 \) kg is attached to the spring, as shown, the spring stretches by 4.8 cm. Calculate \( k \) in SI units.

\[
mg \sin \theta = k \Delta x \Rightarrow k = \frac{mg \sin \theta}{\Delta x} = \frac{2.4 \times 9.8 \times 0.048}{0.048} = 2.45 \text{ N/m}
\]

\boxed{2.45 \text{ N/m}}
A playground is on the flat roof of a city school, 7.00 m above the street below. The vertical wall of the building is 8.00 m high, forming a 1.00 m high railing around the playground. A ball has fallen to the street below, and a passerby returns it by launching it at an angle of 53.0° above the horizontal at a point 27.0 m from the base of the building wall. The ball takes 2.20 s to reach a point vertically above the wall.

(a) [7 pts.] Find the speed at which the ball was launched.

(b) [7 pts.] Find the vertical distance, h, by which the ball clears the wall.

(c) [6 pts.] Find the distance, d, from the wall to the point on the roof where the ball lands.

\[ \theta = 53^\circ, \; y_f = 7, \; D = 27 \text{ m} \]
\[ x = D \cos \theta \theta = 7, \; t = 2.2 \text{ s} \]
\[ u = \frac{D}{t} \cos 53^\circ \times 22 \]
\[ v_y = v_{y,\text{max}} + \frac{1}{2} g t^2 = \left( v_{0,\text{max}} \sin \theta \right) t - \frac{1}{2} g t^2 \]
\[ = 26 \times 4 \sin 53^\circ \times 22 - 4.9 \times (2.2)^2 \]
\[ = 3.58 - 23.7 = 12.1 \text{ m} \]

Substantive height of wall \[ h = 12.1 - 8 = 4.1 \text{ m} \]

Find \( t \) when \( y = 7 \)
\[ 4.9t^2 - 0.34 \sin \theta t + 7 = 0 \]
\[ t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0.34 \sin \theta + \sqrt{0.34 \sin \theta^2 - 4 \times 4.9 \times 7}}{2 \times 4.9} \]
\[ = \frac{16.3}{29.8} \pm \sqrt{\frac{16.3^2}{29.8^2} - \frac{4 \times 7}{29.8}} \]
\[ = 2.8 \text{ s} \]

\[ x = D \cos \theta \theta = 27 + 0 = 34.6 = 27 + 0 \]
\[ \Rightarrow d = 7.6 \text{ m} \]
A physical pendulum consists of a solid sphere and a rod that is rigidly attached to the sphere at point Q, directly opposite to the pivot point P. The sphere has radius $r$ and mass $m$, and the rod has length $L$ and mass $M$.

(a) \[8 \text{ pts.}\] Calculate (in terms of $m$, $M$ and $L$) the moment of inertia of the system about the pivot point $P$. 

(b) \[7 \text{ pts.}\] Assuming small oscillations ($\theta$ small), calculate (in terms of $L$, $m$, $M$ and $g$) the period of oscillation.

\[
I = I_{\text{sph}} + I_{\text{rod}} = I_{P}\text{ about the pivot point}
\]

\[
I_{\text{sph}} = I_{cm} + mr^2 = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2
\]

\[
I_{\text{rod}} = I_{cm} + MD^2, \text{ where } D = \text{dist between cm and pivot}
\]

\[
D = \frac{L}{2} + 2r
\]

\[
I_{\text{rod, pivot}} = \frac{1}{12}ML^2 + M\left(\frac{L}{2} + 2r\right)^2
\]

\[
I_P = \frac{7}{5}mr^2 + M\left[\frac{L^2}{3} + 2Lr + 4r^2\right]
\]

\[
T = \frac{2\pi}{\omega}
\]

\[
\omega = \sqrt{\frac{g}{\frac{7}{5}mL^2 + M\left(\frac{L}{2} + 2r\right)^2}}
\]

where $l = \text{dist. of cm of system from P} = \frac{\sum m_i r_i}{M+m}$

\[
l = \frac{mr + M\left(\frac{L}{2} + 2r\right)}{M+m}
\]

\[
T = \frac{2\pi}{\sqrt{\frac{g}{\frac{7}{5}mr^2 + M\left(\frac{L^2}{3} + 2Lr + 4r^2\right)}}}
\]
A block of mass $m = 0.20$ kg is at rest at the height $h = 4.0$ m on a smooth inclined plane as shown. It slides down onto a frictionless surface and collides inelastically with a second block of identical mass. The velocity of the first block immediately after the collision is $1/3$ its velocity immediately before the collision and is in the same direction. The floor to the right of the second block is rough with a coefficient of kinetic friction $\mu_k = 0.50$. As result, the second block slides a distance $d$ before coming to rest.

(a) \[ 7 \text{ pts.} \] Calculate the speed $v_2$ \((\text{in m/s})\) of the second block immediately after the collision.

(b) \[ 7 \text{ pts.} \] Calculate \((\text{in J})\) the energy lost in the collision.

(c) \[ 6 \text{ pts.} \] Calculate \((\text{in m})\) the distance $d$ that the second block travels before coming to rest.

\[ \begin{align*}
\text{(a)} \quad & 7 \text{ pts.} \quad \text{Calculate the speed } v_2 \text{ (in m/s) of the second block immediately after the collision.} \\
\text{(b)} \quad & 7 \text{ pts.} \quad \text{Calculate (in J) the energy lost in the collision.} \\
\text{(c)} \quad & 6 \text{ pts.} \quad \text{Calculate (in m) the distance } d \text{ that the second block travels before coming to rest.}
\end{align*} \]
In class, Adam did a demonstration in which he shot an arrow of mass $m = 330$ gm horizontally into a block of mass $M = 5.00$ kg, which was suspended a distance $L = 75$ cm below a pivot point $P$. The arrow was embedded in the block and the system swung upwards such that the maximum horizontal displacement $\Delta x$ was observed to be $15$ cm. Treat the arrow as a point mass.

(a) [7 pts.] Calculate the maximum kinetic energy of the system consisting of the arrow embedded in the block.

(b) [7 pts.] Calculate the velocity of the arrow immediately before it struck the block.

(c) [6 pts.] Calculate the angular momentum of the arrow relative to the axis of rotation immediately before impact. Will this be the same as the angular momentum of the system immediately after impact? Explain.