[15 pts.] The position of a particle is given by the function \( x = (2t^3 - 9t^2 + 12) \text{m}, \) where \( t \) is in s.

(a) At what time or times \( v_x = 0 \text{ m/s}? \)
(b) What are the particle's position and acceleration at this time(s)?

\[ a) \quad x = 2t^3 - 9t^2 + 12 \]

\[ v_x = \frac{dx}{dt} = 6t^2 - 18t \]

Find \( v_x = 0 \) \( \Rightarrow \) \( 6t^2 - 18t = 0 \)

\[ 6t(t - 3) = 0 \quad \Rightarrow \quad t = 0_s \text{ and } t = 3_s \]

[All use quadratic formula formula]

\[ t_{1,2} = \frac{18 \pm \sqrt{(18)^2 - 4 \cdot 6 \cdot 0}}{2 \cdot 6} \Rightarrow t = 0_s \text{ and } t = 3_s \]

\[ b) \quad t_1 = 0_s \quad x(0) = 0 - 0 + 12 = 12 \text{ m} \]

\[ a_x = \frac{dv_x}{dt} = 12t - 18 \]

\[ a_x(0) = 12 \cdot 0 - 18 = -18 \text{ m/s}^2 \]

\[ t_2 = 3_s \quad x(3) = 2 \cdot 3^3 - 9 \cdot 3^2 + 12 = -15 \text{ m} \]

\[ a_x(3) = 12 \cdot 3 - 18 = 18 \text{ m/s}^2 \]
A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket’s acceleration?

\[ 0 \leq t < 4 \]

Rocket & Bolt are accelerating upwards

\[ y_1 = y_0 + v_0 t + \frac{1}{2} a t^2 \]

\[ y_1 = 8a \]

\[ V_{f,1} = V_0 + at = 4a \]

\[ 4 \leq t \leq 10 \]

\[ y_f = y_1 + V_{f,1} t - \frac{1}{2} g t^2 \]

The bolt hits the ground \( y_f = 0 \) (6 s later)

\[ 0 = 8a + 24a - \frac{1}{2} g (6)^2 \]

\[ a = \frac{9}{16} g = 5.5 \text{ m/s}^2 \]
The drawing shows four electric charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly away from B) on charge A of 3.0 N. Charge C exerts an attractive force (directly toward C) on charge A of 6.0 N. Finally, charge D exerts an attractive force of 2.0 N on charge A. Assuming that forces are vectors, what are the magnitude and direction of the net force \( \overrightarrow{F_{\text{net}}} \) exerted on charge A?

\[
\overrightarrow{F_B} = -3 \, \hat{x} + 0 \, \hat{y}
\]

\[
\overrightarrow{F_C} = 0 \, \hat{x} - 6 \, \hat{y}
\]

\( \overrightarrow{F_D} \) should be divided up into \( \hat{x} \) and \( \hat{y} \) components

\[
\overrightarrow{F_{Dx}} = 2 \sin \theta_1 = 1.63
\]

\[
\overrightarrow{F_{Dy}} = 2 \cos \theta_1 = -1.16
\]

\[
\overrightarrow{F_{\text{net}}} = \overrightarrow{F_B} + \overrightarrow{F_C} + \overrightarrow{F_D} = -1.37 \, \hat{x} - 7.16 \, \hat{y}
\]

Magnitude:

\[
|\overrightarrow{F_{\text{net}}}| = \sqrt{(-1.37)^2 + (-7.16)^2} \approx 7.3 \, \text{N}
\]

Find direction; Both \( F_{\text{net}} \) and \( F_{\text{net},y} \) are negative:

\[
\tan x = \left( \frac{7.16}{1.37} \right) \Rightarrow x = 79.17^\circ
\]

Find \( \beta \):

\[
\beta = 180^\circ + x = 259.17^\circ \approx 260^\circ
\]

So the direction is \( 260^\circ \), CCW from +x axis.
[20 pts.] A sailor climbs to the top of the mast, 15 m above the deck, to look for land while ship moves steadily forward through calm waters at 4.0 m/s. Unfortunately, he drops his spyglass to the deck below.

(a) Where does the spyglass land with respect to the base of the mast below him?
(b) Where does the spyglass land with respect to a fisherman sitting at rest in his dinghy as the ship goes past?
Assume that the fisherman is even with the mast at the instant the spyglass is dropped.

(a) Since the spyglass and ship have the same velocity in the x-direction, the spyglass will fall at the base of the mast.

\[ Y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2 = h - \frac{1}{2} gt^2 \]
Where \( h = 15 \text{ m} \) is the height of the mast.

The spyglass hit the ground (\( Y = 0 \)) at
\[ t = \sqrt{\frac{2h}{g}} \]

\[ d = v_x \cdot t = v_x \cdot \sqrt{\frac{2h}{g}} = (4.0 \text{ m/s}) \cdot \sqrt{\frac{2(15\text{ m})}{9.8 \text{ m/s}^2}} = 7.00 \text{ m} \]
An amusement park game, shown in the drawing, launches a marble toward a small cup. The marble is placed directly on top of a spring-loaded wheel and held with a clamp. When released, the wheel spins around clockwise at constant angular acceleration, opening the clamp and releasing the marble after making 11/12 revolution. What angular acceleration is needed for the ball to land in the cup?

First find where the marble is when it is released.

\[ \Theta = 2\pi \cdot \left( \frac{11}{12} \right) = \frac{\pi}{6} \text{ rad} = 30^\circ \]

\[ X = \frac{40 \text{ cm} \cdot \sin(30^\circ)}{2} = 0.1 \text{ m} \]

\[ Y = 20 \text{ cm} \cdot \cos(30^\circ) = 0.1732 \text{ m} \]

How far does marble have to travel?

\[ \Delta X = 0.1 \text{ m} + 100 \text{ m} = 1.1 \text{ m} \]

\[ \Delta Y = -0.1732 \text{ m} \]

This is projectile motion. It is launched at 30°, what \( V \) is needed to travel the distance?

\[ Y = Y_0 + V_{yo}t + \frac{1}{2}at^2 \Rightarrow 0 = 0.1732 + V_{yo} - \frac{1}{2}9.8t^2 \]

\[ X = X_0 + V_{xo}t + \frac{1}{2}at^2 \Rightarrow 1.1 = 0 + V_{xo}t + 0 \Rightarrow t = \frac{1.1}{V_{xo}} \]

Express in terms of \( \omega_f \) \[ t = \frac{1.1}{\omega_f \cdot r \cdot \cos\theta} \Rightarrow t = \frac{6.381}{\omega_f} \]

Remove unknown \( t \) from the problem by substituting into \( y \)-equation.

\[ 0 = 0.1732 + \omega_f \cdot 0.2 \text{ m} \cdot \sin(30^\circ) \cdot \frac{6.381}{\omega_f} - \frac{1}{2} \cdot 9.8 \text{ m/s}^2 \left( \frac{6.381}{\omega_f} \right)^2 \]

\[ \Rightarrow \omega_f = 15.637 \text{ rad/s} \]
Now we know $w_f$, $w_0$, and $\Delta \theta$ and we need to know $\alpha = \frac{w_f - w_0}{\Delta t} = \frac{w_f}{\Delta t}$, \(\Rightarrow\) $w_f^2 = w_0^2 + 2\alpha \Delta \theta$

\(\Rightarrow\) $w_f^2 = 2\alpha \Delta \theta \Rightarrow \alpha = \frac{w_f^2}{2\Delta \theta} = \frac{(15.637 \text{ rad/s})^2}{2 \cdot \frac{11}{12} \cdot 2\pi \text{ rad}}$

\(\Rightarrow\) $\alpha = 21.2 \text{ rad/s}^2$