A package of mass $m$ is released from rest at a warehouse loading dock and slides down a 3.0 m high frictionless chute to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass 2$m$, from the bottom of the chute.

(a) [10 pts.] Suppose the packages stick together. What is their common speed after the collision?

(b) [10 pts.] Suppose the collision between the packages is perfectly elastic. To what height does the package of mass $m$ rebound?

\[ mgh = \frac{1}{2}mv_1^2 \]

\[ v_1 = \sqrt{2gh} \]

\[ MV_1 = (m + 2m) v_f \]

\[ v_f = 2.56 \text{ m/s} \]

(b) \[ m v_1 = m v_{1f} + 2m v_{2f} \]

\[ \frac{1}{2}mv_1^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}(2m) v_{2f}^2 \]

\[ y = \frac{m_1}{m_2} = \frac{1}{2} \]

\[ v^2 = v_{1f}^2 + \left(\frac{1}{2}\right)^2 (v - v_{1f})^2 \]

\[ v_{1f} = \frac{v_y \pm v \sqrt{v_y^2 - 4y}}{2y} = \frac{v \left[ \frac{y + 1}{y + 1} \right]}{y + 1} = v - \frac{y}{3} \]

\[ h_f = \frac{v_{1f}^2}{2g} = \frac{h_0}{9} = .33 \text{ m} \]
A Porsche 944 Turbo has a rated engine power of 217 hp. Thirty percent (30%) of the power is lost in the drive train, and 70% reaches the wheels. The total mass of the car and driver is 1480 kg, and two-thirds of the weight is over the drive wheels.

(a) [8 pts.] What is the maximum acceleration of the Porsche on a concrete surface where \( \mu_s = 1.00 \)?

(b) [8 pts.] If the Porsche accelerates at \( a_{\text{max}} \), what is its speed when it reaches maximum power output?

(c) [4 pts.] How long does it take the Porsche to reach the maximum power output?

\[ f_s = \mu_s N = \mu_s \frac{2}{3} mg \]

\[ F_x = f_s = ma \]

\[ a_{\text{max}} = \frac{f_s}{m} = \frac{\mu_s \frac{2}{3} mg}{m} = \mu_s \frac{2}{3} g = 1 \cdot \frac{2}{3} \cdot 9.8 = 6.53 \text{ m/s}^2 \]

\[ P = F \cdot v \]

\[ P_{\text{max}} = F_{\text{max}} \cdot v_{\text{max}} \]

\[ v_{\text{max}} = \frac{P_{\text{max}}}{F_{\text{max}}} = \frac{0.7 \cdot 217 \text{ hp} \cdot 746 \text{ W}}{1480 \cdot 6.53} = 11.7 \text{ m/s} \]

\[ v_f = v_i + a \Delta t, \quad v_i = 0 \]

\[ \Delta t = \frac{v_f}{a} = \frac{11.7}{6.53} = 1.79 \text{s} \]
A cube of mass \( m \) slides without friction at speed \( v_0 \). It undergoes a perfectly elastic collision with the bottom tip of a rod of length \( d \) and mass \( M = 2m \). The rod is pivoted about a frictionless axle through its center, and is at rest. What is the cube’s velocity, both speed and direction, after the collision?

\[
\begin{align*}
F_0 &= F_f \Rightarrow \frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2 \\

L_i &= L_f \Rightarrow m v_0 \left( \frac{d}{2} \right) = m v_f \left( \frac{d}{2} \right) + I \omega \\
I &= \frac{1}{12} M d^2 = \frac{1}{12} (2m) d^2 = \frac{1}{6} m d^2 \\
m v_0 \frac{d}{2} &= m v_f \frac{d}{2} + \frac{1}{6} m d^2 \omega \Rightarrow \frac{v_0}{2} = \frac{v_f}{2} + \frac{wd}{3} \Rightarrow \frac{v_0}{2} = \frac{v_f}{2} + \frac{wd}{3} \\
\omega &= \frac{3(v_0 - v_f)}{d} \\
\Rightarrow \frac{1}{2} m v_0^2 &= \frac{1}{2} m v_f^2 + \frac{1}{2} \left( \frac{1}{6} m d^2 \right) \left( \frac{3(v_0 - v_f)}{d} \right)^2 \Rightarrow v_0^2 = v_f^2 + \frac{3}{2} (v_0 - v_f)^2 \\
v_0^2 = v_f^2 + 3(v_f - 2v_0 v_f + v_0^2) \Rightarrow \frac{v_0^2}{2} - 3v_0 v_f + \frac{3}{2} v_f^2 = 0 \\
v_0^2 - 6v_0 v_f + 3v_f^2 = 0 \Rightarrow v_f^2 - \frac{6}{5} v_0 v_f + \frac{1}{5} v_0^2 = 0 \\
\Rightarrow v_f = -\frac{6v_0 + \sqrt{(6v_0)^2 - 4(\frac{3}{5}v_0^2)}}{2(\frac{3}{5})} = \frac{3}{5} v_0 + \frac{2}{5} v_f \\
\Rightarrow v_f = \frac{v_0}{5}, \text{ to the right} \\
\Rightarrow v_f = \frac{v_0}{5}, \text{ to the right} \\
\Rightarrow v_0 = v_f \text{ means it has not hit the rod} \\
\Rightarrow v_f = \frac{v_0}{5}, \text{ to the right} \\
\Rightarrow v_f = \frac{v_0}{5}, \text{ to the right} 
\end{align*}
\]
While visiting Planet Physics, you toss a rock straight up at 11 m/s and catch it 2.5 s later. While you visit the surface, your cruise ship orbits at an altitude equal to the planet’s radius every 230 min.

(a) \(\text{[pts.]}\) What is the mass of Planet Physics?

(b) \(\text{[pts.]}\) What is the radius of Planet Physics?

- First use kinematics to find \(g\) (the free-fall acceleration near the surface of the planet):
  \[V_f = V_0 - gt \implies g = \frac{V_0 - V_f}{t}\]

  We know for the trajectory: \(V_f = -V_0\)

  \[g = \frac{V_0 - (-V_0)}{t} = \frac{2V_0}{t} = \frac{2\,(11\,\text{m/s})}{2.5\,\text{s}} = 8.80\,\text{m/s}^2\]

- Relate the free-fall acceleration to the mass, \(M\), of the planet and its radius, \(R\), using Newton’s law of gravitation.
  \[-mg = -\frac{GMm}{R^2} \implies g = \frac{GM}{R^2}\]

- We need one more equation relating \(M\) and \(R\).
  - Apply Kepler’s third law to the trajectory of the orbiting ship
    \[T^2 = \frac{4\pi^2}{GM}r^3\] where \(r = 2R\)
\[ T^2 = \frac{32 \pi^2}{GM} R^3 \]  
\[ M = \frac{32 \pi^2 R^3}{GT^2} \]

- Using the above expression and \( g = \frac{GM}{R^2} \) (or \( M = \frac{R^2 g}{G} \))

\[ \frac{R^2}{g} = \frac{32 \pi^2 R^3}{GT^2} \]

\[ R = \frac{gT^2}{32 \pi^2} = \frac{(8.80 \text{ m/s}^2)(230 \text{ min} \times 60 \text{ sec/min})^2}{32 \pi^2} \]

\[ R = 5.31 \times 10^6 \text{ m} \]

\[ M = \frac{R^2 g}{G} = \frac{(5.31 \times 10^6 \text{ m})^2 (8.80 \text{ m/s}^2)}{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)} = 3.71 \times 10^{24} \text{ kg} \]
A spring is standing upright on a table with its bottom end fastened to the table. A block is dropped from a height 3.0 cm above the top of the spring. The block sticks to the top end of the spring and then oscillates with an amplitude of 10 cm. What is the oscillation frequency?

\[
\omega = \sqrt{\frac{k}{m}} \quad \text{Equilibrium Point: } k\Delta L = mg
\]
\[
\Rightarrow \omega = \sqrt{\frac{g}{\Delta L}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta L}}
\]
\[
mg\Delta h + mg\Delta L + mgA = \frac{1}{2} k(\Delta L + A)^2 = \frac{mg}{2\Delta L} (\Delta L + A)^2
\]
\[
\Rightarrow (\Delta l)^2 + 2h\Delta L - A^2 = 0
\]
\[
\Delta L = -2h + \sqrt{4h^2 + 4A^2} = \sqrt{h^2 + A^2} - h
\]
\[
\Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{h^2 + A^2} - h} = \frac{1}{2\pi} \sqrt{\frac{g}{2.0cm^2 + h(3.0cm)^2} - 3.0cm}
\]
\[
\Rightarrow f = 1.83 \text{ Hz}
\]