In an amusement ride called The Roundup, passengers stand inside a 16 m diameter rotating ring. After the ride has acquired sufficient speed, it tilts into a vertical plane, as shown in the drawing.

(a) Suppose the ring rotates once every 4.5 s. If a rider’s mass is 55 kg, with how much force does the ring push on her at the top of the ride? At the bottom of the ride?

(b) What is the longest rotation period of the wheel that will prevent the rider from falling off at the top?

\( a = -\omega^2 r \)

\( a = -\omega^2 r \)

At top:

\[ \begin{align*}
\Sigma F_y &= -N - mg = ma \\
\Sigma F_y &= -N + mg = m\omega^2 r \\
N &= m\omega^2 r - mg = (55 \text{ kg})(\left(\frac{2\pi \text{ rad}}{4.5 \text{ s}}\right)^2 \cdot 9.8 \text{ m/s}^2) - 9.8 \text{ m/s}^2 \\
N &= 319 \text{ N}
\end{align*} \]

At bottom:

\[ \begin{align*}
\Sigma F_y &= N - mg = ma \\
\Sigma F_y &= N - mg = m\omega^2 r \\
N &= m\omega^2 r + mg = (55 \text{ kg})(\left(\frac{2\pi \text{ rad}}{4.5 \text{ s}}\right)^2 \cdot 9.8 \text{ m/s}^2 + 9.8 \text{ m/s}^2) \\
N &= 1400 \text{ N}
\end{align*} \]

(b) \( N = 0 \) at top

\[ \begin{align*}
\Sigma F_y &= -mg = ma \\
- mg &= m\omega^2 r \\
\omega &= \sqrt{\frac{g}{r}} \\
T &= \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r}{g}} = 2\pi \sqrt{\frac{8\text{ m}}{9.8 \text{ m/s}^2}} = 5.68 \text{ s}
\end{align*} \]
(25 pts.) Ann (mass 50 kg) is standing at the left end of a 15 m long, 500 kg cart that has frictionless wheels and rolls on a frictionless track. Initially, both Ann and the cart are at rest. Suddenly, Ann starts running along the cart at a speed of 5.0 m/s relative to the cart. How far will Ann have run relative to the ground when she reaches the right end of the cart?

\[ v' = 5.0 \text{ m/s}, \ m = 50 \text{ kg}, \ M = 500 \text{ kg}, \ d = 15 \text{ m}, \ V'_M = 0 \]

\[ mv'_m = -MV'_M, \ V'_m = V_m - V'_M \]

\[ \Rightarrow V_m = \frac{M}{m + M} \cdot V'_m \]

\[ \Delta t = \frac{d}{V_m} = 3 \text{ s} \]

\[ \Delta x = V_m \Delta t = \frac{M}{m + M} \cdot V'_m \frac{d}{V'_m} = \frac{Md}{M + m} = 13.64 \text{ m} \]
[25 pts.] It has been a great day of new, frictionless snow. Julie starts from rest at the top of the 60° slope as shown in the drawing. At the bottom, a circular arc carries her through a 90° turn, and she then launches off a 3.0 m high ramp. How far horizontally is her touchdown point from the end of the jump?

\[ E_i = E_f \quad \text{(first energy conservation)} \]

\[ mg h_0 = \frac{1}{2} m v^2 + mg h \]

\[ v = \sqrt{2g (h_0-h)} = 20.8 \text{ m/s} \]

Now kinematics to determine \( \Delta x \):

\[ y_f = y_i + v_i y t - \frac{1}{2} g t^2 \]

\[ 0 = 0 + Vs \sin(30°) - \frac{1}{2} g t^2 \]

\[ \text{Quadratic} \quad \text{for} \quad \text{(take positive root)} \]

\[ t = \frac{v_s \sin(30°)}{g} + \sqrt{\left(v_s \sin(30°)\right)^2 + 2g y_i} \]

\[ t = 2.38 \text{ s} \]

\[ \Delta x = (v_c x) t = v \cos(30°) t = 42.7 \text{ m} \]
A merry-go-round is a common piece of playground equipment. A 3.0 m diameter merry-go-round with a mass of 250 kg is spinning at 20 rpm. John runs tangent to the merry-go-round at 5.0 m/s, in the same direction that it is turning, and jumps onto the outer edge. John’s mass is 30 kg. What is the merry-go-round’s angular velocity, in rpm, after John jumps on?

**Conservation of angular momentum**

\[
L_0 = L_f
\]

\[
\uparrow \text{ after John jumps on}
\]

before John jumps on

Merry-go-round ⇒ Disk

\[
I_{disk} = \frac{1}{2} MR^2
\]

\[
L_0 = I_{disk} \omega_i + M_J VR
\]

\[
L_f = I_{disk} + M_J R^2
\]

John is considered to be a point mass a distance \( R \) from axis.

\[
L_f = I_f \omega_f
\]

(first convert \( \omega_i \) to rad/s)

\[
\omega_f = \frac{\frac{1}{2} MR^2 \omega_i + M_J VR}{\frac{1}{2} MR^2 + M_J R^2}
\]

\[
2.33 \text{ rad/s} \Rightarrow 22.3 \text{ RPM}
\]
A particle of mass $m$ is at a distance $x$ along the axis of a very thin ring of mass $M$ and radius $R$.

(a) \[ \text{Calculate the gravitational potential energy of these two masses.} \]

(b) \[ \text{Use what you know about the relationship between force and potential energy to find the magnitude and direction of the gravitational force on } m \text{ when it is at position } x. \]

\[ dU = -\frac{GmM}{r} \]
\[ r = \sqrt{x^2 + R^2} \]
\[ U = \int dU = -\int \frac{GmM}{r} = -\int \frac{GmM}{2\pi R} d\theta = -\frac{GmM}{\sqrt{x^2 + R^2}} \]

\[ F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z} \]

Clearly \[ \frac{\partial U}{\partial y} = \frac{\partial U}{\partial z} = 0 \text{ since } U \text{ doesn't depend on them} \]

\[ F_x = -\frac{\partial}{\partial x} \left( -\frac{GmM}{\sqrt{x^2 + R^2}} \right) = \frac{GmMx}{(x^2 + R^2)^{3/2}} \]

\[ \text{Negative sign indicates it is in the negative } x \text{ direction.} \]
[25 pts.] A circular hoop of mass $M$ and radius $R$ is pivoted on an axle passing through one edge, as shown in the drawing. Find an expression for the frequency of small oscillations.

\[ C = I \alpha, \quad C = Mg \sin \theta \approx MgR \theta \]

\[ \sqrt{\frac{C}{I}} = \omega = \sqrt{\frac{MgR}{I}}, \quad I_0 = MR^2 \text{ (for hoop)} \]

\[ I = I_0 + Md^2 = 2MR^2 \text{ (parallel axes + co-rot.)} \]

\[ \omega = \sqrt{\frac{g}{2R}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{2R}} \]
A water tank of height $h$ has a small hole at height $y$. The water is replenished to keep $h$ from changing. The water squirting from the hole has range $x$. The range approaches zero as $y \to 0$ because the water squirts right onto the table. The range also approaches zero as $y \to h$ because the horizontal velocity becomes zero. Thus, there must be some height $y$ between 0 and $h$ for which the range is a maximum.

(a) \( \left\{ \begin{array}{l} \rho g h_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 \\
\rho = \rho = \rho_{\text{air}} \\
h_1 = h \\
h_2 = y \\
v_1 = 0 \text{ (water is replenished)} \end{array} \right. \Rightarrow v_2 = \sqrt{2 g \sqrt{h-y}} \]

(b) \( y_f = y_i + (v_i) y \Delta t + \frac{1}{2} a_y \Delta t^2 \)

\( \Delta t = \sqrt{\frac{2 y}{g}} \)

\( x_f = x_i + (v_i) x \Delta t + \frac{1}{2} a_x \Delta t^2 \)

\( x_f = 0 + (v_x) x \Delta t + 0 = v \sqrt{\frac{2 y}{g}} \Rightarrow y \sqrt{h-y} \)

(c) To find $x_{\text{max}}$:

\( \frac{dx}{dy} = 0 \)

\( \frac{d}{dy} \left( 2 (hy-y^2)^{1/2} \right) = \frac{h \cdot 2y}{\sqrt{hy-y^2}} = 0 \)

\( \Rightarrow h - 2y = 0 \)

\( \Rightarrow y = \frac{h}{2} \)

\( \Rightarrow x_{\text{max}} = 2 (hy-y^2)^{1/2} = 2 \sqrt{\frac{h^2}{4}} = \frac{h}{2} \)

Note: You need both $y$ and $x_{\text{max}}$ for full credit!
Two masses are hanging from a steel wire (as shown in the drawing). The mass of the 8.0 m long wire is 60.0 g. A wave pulse travels along the wire from point 1 to point 2 in 24.0 ms. What is mass m?

\[ v = \sqrt{\frac{T}{\mu}} \quad T = \text{tension} \]
\[ v = \frac{d}{t} = \frac{4 \text{m}}{24 \text{ms}} = 166.7 \text{ m/s} \]
\[ \mu = \frac{M}{L} = \frac{60 \text{g}}{8 \text{m}} = 0.0075 \text{ kg/m} \]
\[ T = 0.75 \mu = 208 \text{N} \]

Look at one side:

\[ \Sigma F_x = T - T_w \cos 40^\circ = 0 \quad \Rightarrow T_w = \frac{T}{\cos 40^\circ} \]
\[ \Sigma F_y = T_w \sin 40^\circ - mg = 0 \]
\[ \Rightarrow \frac{T}{\cos 40^\circ} \sin 40^\circ - mg = T \sin 40^\circ - mg = 0 \]
\[ \Rightarrow m = \frac{T \sin 40^\circ}{g} = \frac{208 \frac{\text{N} \cdot \text{m}}{\text{s}^2}}{9.8} = 17.8 \text{ kg} \]

**Alt 2 (Assuming T is constant throughout the wire)**

As in Alt 1: \( T = 208 \text{N} \)

\[ 2T \sin 40^\circ = 2mg \]
\[ T = \frac{mg}{\sin 40^\circ} \quad \Rightarrow m = \frac{T \sin 40^\circ}{g} = 13.7 \text{ kg} \]