A load of bricks is being lifted by a crane at the steady velocity of 16 ft/sec, but 20 ft above the ground one brick falls off.

(a) What is the greatest height the brick reaches above the ground?
(b) How long does it take to reach the ground?
(c) What is its speed just before it hits the ground?

\[ y = 20 \text{ ft} \]

\[ y = 0 \]

\[ \text{The position of the brick at time } t \text{ is given by} \]

\[ y(t) = y_0 + v_0 t + \frac{1}{2} gt^2, \]

 WHERE \( t = 0 \) IS THE MOMENT AT WHICH THE BRICK DROPS FROM THE CRANE.

AT \( t=0 \), \( y_0 = 20 \text{ ft} \) \( \text{(initial position)} \)

AND \( v_0 = 16 \text{ ft/s} \) \( \text{(initial velocity)} \);

JUST BEFORE THE BRICK FALLS OFF IT HAS THE SAME VELOCITY AS THE REST OF THE LOAD.

a) To find the greatest height attained, we maximize the function \( y(t) \):

\[ \frac{dy}{dt} = v(t) = v_0 + gt = 0 \text{ so } t = \frac{-v_0}{g} = \frac{-16 \text{ ft/s}}{-32.2 \text{ ft/s}^2} = 0.50 \text{ s}. \]

AT THIS TIME THE HEIGHT OF THE BRICK IS

\[ y_{\text{max}} = 20 \text{ ft} + (16 \text{ ft/s})(0.50 \text{ s}) + \frac{1}{2} (-32.2 \text{ ft/s}^2)(0.50 \text{ s})^2 = 24 \text{ ft}. \]

b) The brick reaches the ground when \( y(t) = 0 \), that is

\[ 0 = y_0 + v_0 t + \frac{1}{2} gt^2 \Rightarrow t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}g\right)y_0}}{g} \]

\[ t = \frac{-16 \pm \sqrt{16^2 - (-2 \cdot 32.2 \cdot 20)}}{-32.2} \text{ s} = 1.72 \text{ s or -0.72 s}. \]

WE ABANDON THE SECOND SOLUTION AS BEING UNPHYSICAL. \( t = 1.72 \text{ s}. \)

c) The velocity of the brick is \( v(t) = v_0 + gt \) so at \( t = 1.72 \text{ s}, \)

\[ v(1.72 \text{ s}) = 16 \text{ ft/s} - (32.2 \text{ ft/s}^2)(1.72 \text{ s}) = -39 \text{ ft/s} \] \( \text{so} \)

\[ \text{SPEED} = 39 \text{ ft/s}. \]
A ball is thrown down vertically with an initial speed of 20 m/s from a height of 60 m.

10 (a) What will be its speed just before it strikes the ground?

10 (b) How long will it take for the ball to reach the ground?

10 (c) What would be the answers to (a) and (b) if the ball were thrown directly up from the same height and with the same initial speed?

(y increases upward)

**FOR PARTS (a) AND (b),** \( y_0 = 60 \text{ m}, \ V_0 = -20 \text{ m/s} \).

a) **If \( V_0 \) is the speed just before striking the ground,**

then \( V_0^2 - V_0^2 = 2g(0 - 60\text{ m}) \)

\( V_0^2 = (-20 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(-60\text{ m}) = 158 \cdot 10^3 \text{ m}^2/\text{s}^2 \)

so the required speed is \( \sqrt{158 \cdot 10^3 \text{ m}^2/\text{s}^2} = 40 \text{ m/s} \)

b) \( V_0 - V_0 = gt \) so \( t = \frac{V_0-V_0}{g} = \frac{-40\text{ m/s} - (-20\text{ m/s})}{-9.8 \text{ m/s}^2} = 2.0 \text{ s} \).

The ball hits the ground after \( 2.0 \text{ s} \).

(We have chosen \( V_0 = -40\text{ m/s} \) since \( V_0 = +40\text{ m/s} \) would be unphysical.)

c) **This time \( V_0 = +20 \text{ m/s} \), so**

\( V_0^2 = V_0^2 + 2g(0 - 60\text{ m}) = (20\text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(60\text{ m}) \)

i.e. it's the same thing.

\[ |V_0| = \boxed{40 \text{ m/s}}. \]

\( V_0 - V_0 = gt \) so \( t = \frac{V_0-V_0}{g} = \frac{-40\text{ m/s} - (20\text{ m/s})}{-9.8 \text{ m/s}^2} = \boxed{6.1 \text{ s}} \).