A block of mass \( m \) slides from rest down a frictionless circular track, as shown. It strikes a second block, of mass \( 3m \), exactly at the bottom, and they stick together. (a) Find the largest value of \( \theta \), the angular position after the collision. (b) When \( \theta \) is at its largest value, as in (a), find the normal force the track exerts on the block.

Energy conservation for \( m \):

\[
(+) 4 \quad V_o = \sqrt{2gR} = \sqrt{gR}
\]

(\( V_o \): velocity of \( m \) before it hits \( 3m \)).

(a) Momentum conservation at \( \theta = 0 \):

\[
(+) 8 \quad mV_o = 4mV_{\text{bottom}} \implies V_{\text{bottom}} = \frac{1}{4} \sqrt{gR} : \text{velocity of } 4m \text{ at } \theta = 0.
\]

Energy conservation for \( 4m \):

\[
(+) 6 \quad \frac{1}{2} 4m V_{\text{bottom}}^2 = 4mgR(1 - \cos \Theta_{\text{max}})
\]

\[
\frac{1}{32} = 1 - \cos \Theta_{\text{max}} \implies \cos \Theta_{\text{max}} = \frac{31}{32} \implies \Theta_{\text{max}} = 14.4^\circ
\]

Remark: This problem can also be solved by invoking angular momentum conservation at \( \theta = 0 \), and conservation of rotational kinetic energy plus potential energy after collision.

(b) The centripetal acceleration is zero for \( \theta = \Theta_{\text{max}} \), since \( \dot{\theta} = 0 \) there. This implies that

\[
(+) 7 \quad N = W \cos \Theta_{\text{max}} = 4mg \cos \Theta_{\text{max}}
\]

\[
= \frac{31}{8} mg = 3.88 \text{ mg}
\]