PROBLEM 2

(a) Find the y coordinate of the center of mass of the object shown. The object is a sheet of metal of density \( \rho \) and thickness \( t \).

(b) Find the mass of this object.

**Solution:**

(a) \( y_{cm} = \frac{\int y \, dm}{\int dm} \)

where \( dm = \rho t (x_0 - x) \, dy = \rho t \left( x_0 - \frac{y}{\sqrt{a}} \right) \, dy \)

Thus \( \int y \, dm = \rho t \int_{ax_0^4}^{ax_0^4} y \left( x_0 - \frac{y}{\sqrt{a}} \right) \, dy = \frac{\rho t a^2 x_0^9}{18} \)

\( \int dm = \rho t \int_{0}^{ax_0^4} \left( x_0 - \frac{y}{\sqrt{a}} \right) \, dy = \frac{\rho at x_0^5}{5} \)

Therefore, we find \( y_{cm} = \frac{\int y \, dm}{\int dm} = \frac{\frac{\rho t a^2 x_0^9}{18}}{\frac{\rho at x_0^5}{5}} = \frac{\frac{5-a}{18} x_0^4}{5} \)

(b) \( M_{total} = \int dm = \rho t \int_{0}^{ax_0^4} \left( x_0 - \frac{y}{\sqrt{a}} \right) \, dy = \frac{\rho at x_0^5}{5} \)

\( \bar{y} = \int_{0}^{x_0} \frac{y \, dx}{\int_{0}^{x_0} y \, dx} = \int_{0}^{x_0} a x^4 \, dx = \frac{\rho at x_0^5}{5} \)