A massless Hooke's Law spring has an unstretched length of 2.25 m. When a 10.0 kg mass is placed on it, and slowly lowered until the mass is at rest, the spring is squeezed to 2.00 m length. The same 10.0 kg mass is dropped from a height of 6.25 m above the spring.

(a) What is the maximum value of the compression of the spring? (Numerical answer)

(b) What is the velocity of the block after the spring has been compressed 0.25 m? (Numerical answer). Find max. compression ($x$)

\[ F = kx \]

\[ x = 0.25 \text{ m} \]

\[ m = 10 \text{ kg} \]

\[ F = mg \]

\[ mg = -kx \]

\[ \frac{mg}{-x} = k \]

\[ 10 \times (9.8 \times 0.25) = -k \]

\[ k = 392 \text{ N/m} \]

\[ \sqrt{k} = 19.8 \text{ N/m} \]

\[ \text{for } k \]

\[ x = \frac{12.5 mg}{k} - \frac{2mgx}{k} = 0 \]

\[ x^2 - 12.5 \frac{mg}{k} x - \frac{2mgx^2}{k} = 0 \]

\[ k = 392 \text{ N/m} \]

\[ mg = 98 \]

Substituting and cancelling gives:

\[ x^2 - 12.5x - 3.125 = 0 \]

\[ x_1 = 2.64 \text{ m} \]

\[ x_2 = -1.54 \text{ m} \text{ extraneous} \]

\[ x = 2.64 \text{ m} \]

All drawings must be labeled.
Problem 3 continued

(b) Find \( v_{\text{block}} \) when \( x = 0.25 \text{ m} \)

\[ E_{A'} = E_{B'} \]

\[ m g H' = \frac{1}{2} m v_{B'}^2 + \frac{1}{2} k x^2 \]

\[ H' = 6.25 + x \]

\[ x = 0.25 \text{ m (given in statement of problem)} \]

\[ \therefore H' = 6.5 \text{ m} \]

\[ m g (6.5) = \frac{1}{2} m v_{B'}^2 + \frac{1}{2} k (0.25)^2 \]

(from part (a))

\[ m = 10 \text{ kg} \]

\[ k = 392 \text{ N/m} \]

\[ m g = 98 \text{ N} \]

\[ (98N)(6.5m) = \frac{1}{2} (10) v_{B'}^2 + \frac{1}{2} (392) (0.25)^2 \]

\[ 637J = 5 v_{B'}^2 + 12.25J \]

\[ v_{B'} = \sqrt{\frac{1}{5} (637-12.25)J} \]

\[ v_{B'} = 11.2 \text{ m/s} \]

\[ +10 \text{ for } v_{B'} \]

\[ +10 \text{ for } v_{B'} \]
Problem 3 still continued

An alternate method

A method for finding \( \dot{a} \) is the same as in the first method.

Find velocity at \( B \):

\[ \frac{\text{L}}{g} \dot{a} = \frac{\text{L}}{g} a \dot{\gamma} = \frac{\text{L}}{g} \gamma = 6.25 \text{m} \]

\[ a = g \]

\[ \gamma = 0 \]

\[ \frac{\text{L}}{g} \dot{a} = ? \]

\[ \frac{\text{L}}{g} \dot{a} = g (6.25 \text{m}) \]

\[ \dot{a} = \sqrt{g (6.25 \text{m})} \]

\[ \dot{a} = 11.07 \text{ m/s} \]

Now,

\[ \frac{1}{2} m \ddot{\gamma}_{a}^{2} + m g h = \frac{1}{2} m \ddot{\gamma}_{B}^{2} + \frac{1}{2} k x^{2} \]

\[ h = x \]

\[ m g = 96 N \]

\[ m = 10 \]

\[ k = 392 \]

\[ 5 \ddot{\gamma}_{a}^{2} + 98 x = 5 \ddot{\gamma}_{B}^{2} + 196 x^{2} \]

but \( \ddot{\gamma}_{B} = 0 \)

\[ \frac{\text{L}}{g} \dot{a} = 11.07 \text{ m/s} \]

so:

\[ 5 \ddot{\gamma}_{a}^{2} + 98 x = 196 x^{2} \]

\[ \ddot{\gamma}_{a} = \ddot{\gamma}_{B} = 11.07 \]

\[ 612.7 + 98 x = 196 x^{2} \]

\[ 612.7 + 98 x = 196 x^{2} \]

\[ 3 \ddot{\gamma}_{B} + 6.5 x = x^{2} \]

\[ x^{2} - 6.5 x - 312.6 = 0 \]

which is identical to the quadratic equation in the first method.

\[ x = 2.64 \text{ m} \]
Problem 3 again still continued

(An alternate method part 6)

6) Find \( \omega B' \) at point B'.

From part 5:

\[ \omega_B = 11.07 \text{ m/s} \]  

In 3 sig. figs, this is 11.1 m/s. This is \textbf{NOT} the answer. Only 11.2 m/s will be accepted as correct.

\[ h = x = 0.25 \text{ m} \]
\[ m = 10 \text{ kg} \]
\[ mg = 98 \text{ N} \]
\[ k = 392 \text{ N/m} \]

\[ \begin{align*}
\frac{1}{2} (10)(11.27)^2 + (98)(0.25) &= \frac{1}{2} k \omega_B^2 + \frac{1}{2} (392)(0.25)^2 \\
612.7 + 24.125 &= 5 \omega_B^2 + 12.25 \omega_B \\
636.825 &= 5 \omega_B^2 + 12.25 \omega_B \\
\omega_B^2 &= \frac{1}{5} (624.9) \\
\omega_B &= 11.2 \text{ m/s} \\
\end{align*} \]

+10 for \( \omega_B' \)