A small mass, \( m_1 \), slides in a completely frictionless spherical bowl. \( m_1 \) starts at rest at a height \( h = \frac{1}{2} R \) above the bottom of the bowl. When it reaches the bottom of the bowl it strikes a mass \( m_2 \), where \( m_2 = 3m_1 \), in a completely elastic collision.

(a) Calculate the height that \( m_2 \) moves up the bowl after the collision (measured vertically from the bottom of the bowl.)

(b) Calculate the height that \( m_1 \) moves up the bowl after the collision.

**Diagram:**

- \( m_1 \)
- \( m_2 \)
- \( v_1 \rightarrow v_2 \)
- \( m_1 \) and \( m_2 \) are colliding.

---

**Equations:**

1. \( m_1 v_1 + m_2 v_2 = M_1 V_0 \)
2. \( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 V_0^2 \)

where \( V_0 \) is given by \( \frac{1}{2} m_1 x^2 = m_1 gh \)

\( V_0 = \sqrt{2gh} = \sqrt{2gh} \frac{1}{2} R = \sqrt{gR} \)

---

**Steps:**

1. \( m_2 v_2 = m_1 (V_0 - v_1) \)
2. \( m_2 v_2^2 = m_1 (V_0^2 - v_1^2) = m_1 (V_0 - v_1)(V_0 + v_1) \)
3. \( v_2 = V_0 + v_1 \)
4. \( m_1 v_1 + m_2 v_2 = m_1 V_0 \)
5. \( v_1 = \frac{m_1 - m_2}{m_1 + m_2} V_0 \), \( v_2 = \frac{2m_1}{m_1 + m_2} V_0 \)
By conservation of energy, (let \( h_1 \) and \( h_2 \)) be the final heights \( m_1 \) and \( m_2 \) can reach, then:

\[
m_1gh_1 = \frac{1}{2}m_1v_1^2, \quad \Rightarrow \quad h_1 = \frac{v_1^2}{2g}
\]

\[
m_2gh_2 = \frac{1}{2}m_2v_2^2, \quad \Rightarrow \quad h_2 = \frac{v_2^2}{2g}
\]

So:

\[
h_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2 \cdot \frac{V_0^2}{2g}
\]

\[
h_2 = \left( \frac{2m_1}{m_1 + m_2} \right)^2 \cdot \frac{V_0^2}{2g}
\]

By:

\[
m_2 = 3m_1, \quad V_0 = \sqrt{2gh} = \sqrt{gR}
\]

So:

\[
h_1 = \frac{R}{8}
\]

\[
h_2 = \frac{R}{8}
\]