The cross hatched figure shown has a density $\rho$, and a thickness $t$. The quantities $b$ and $a$ are positive constants. Calculate the moment of inertia of this object for rotation about the $y$-axis. Be sure to put your answer in proper finished form.

\[ I_y = \frac{m \int r^2 \, dm}{\int dm} \]

\[ dm = \rho t \, dy \, dx = \rho t (b-ax) \, dx \]

\[ \int_0^b \int_0^t x^2 (b-ax) \, dx = \rho t \int_0^b \left( bx^2 - ax^3 \right) \, dx \]

\[ = \rho t \left[ \frac{bx^3}{3} - \frac{ax^4}{4} \right]_0^b = \rho t \left[ \frac{b^4}{3} - \frac{b^3}{4} \right] \]

\[ \int_0^a (b-ax) \, dx = \rho t \left[ bx - \frac{ax^2}{2} \right]_0^b = \rho t \left[ \frac{b^2}{2} - \frac{b^2}{2} \right] \]

\[ I_y = \frac{\rho t \left[ \frac{b^4}{3} - \frac{b^3}{4} \right]}{\rho t \left[ \frac{b^2}{2} - \frac{b^2}{2} \right]} = \frac{b^2}{a^2} \cdot \frac{1}{\frac{1}{2}} = \frac{1}{6} M \frac{b^2}{a^2} \]