The cross-hatched area shown is an object bounded by a curve given by \( y = A - bx^2 \) and the x-axis. It is a plate of material of density \( \rho \) and thickness \( t \).

(a) Calculate the y coordinate of its center of mass.

Set up the integral only for part (a).

(b) Calculate its moment of inertia for rotation about the y axis. Express in proper form.

(c) Calculate its moment of inertia for rotation about the line \( x = 10 \).

\[
\begin{align*}
\text{I}_{cm} &= \frac{\int y \, dm}{\int dm} \\
\text{I}_{cm} &= \frac{\int_A^0 y \rho \sqrt{A-y} \, dy}{\int_A^0 \rho \sqrt{A-y} \, dy} \\
\text{I}_{cm} &= \frac{A \sqrt{A-y} \, dy}{\int_A^0 \sqrt{A-y} \, dy} \\
\text{I} &= M \int \frac{r^2 \, dm}{\int dm} = M \int x^2 \, dm \\
\text{I} &= 2 \int_0^{\frac{A}{\sqrt{3}}} x^2 \left( \frac{A-x^2}{2} \right) \rho \, dx \\
\text{I} &= M \left[ \frac{Ax^3}{3} - \frac{bx^5}{5} \right]_{\frac{A}{\sqrt{3}}}^{\frac{A}{\sqrt{6}}} \\
\text{I} &= M \frac{A^2}{5} \left[ \frac{1}{2} - \frac{1}{2} \right] \\
\text{I} &= M \frac{A^2}{5} \cdot \frac{4}{15} = \frac{MA^2}{5b} \\
\text{I}_{cm} &= \text{I} + M(100) \\
\text{I}_{cm} &= \frac{MA}{5b} + M(100)
\end{align*}
\]