2. Consider a planet where the density varies according to the relationship 
\[ \rho = \rho_0 \left(1 - \alpha R^3 \right) \], where \( \rho_0 \) is the density at the center, \( \alpha \) is a constant, 
and \( R \) is the distance from the center. The radius of the planet is \( R_0 \).

(a) Calculate the mass of the planet.
(b) Calculate the value of \( \ddot{\mathbf{g}} \) at a point below the surface of the planet, 
which is a distance \( R_0 / 3 \) from the center of the planet.

\[ \begin{align*}
(a) \quad M &= \int_{\text{volume}} dM = \int_{R=0}^{R=R_0} dM(R) = \int_{R=0}^{R=R_0} \rho(R) dV = \int_{R=0}^{R=R_0} \rho(R) 4\pi R^2 dR \\
&= 4\pi \rho_0 \left( \frac{R_0^3}{3} - \frac{\alpha R_0^4}{4} \right) \int_{R=0}^{R=R_0}
\end{align*} \]

(b) \( \ddot{\mathbf{g}} = \mathbf{g}(-\hat{r}) = -g\hat{r} \) (\( \hat{r} \) is unit vector directed radially outward from center of earth.

We know the total contribution of the region to the \( \ddot{\mathbf{g}} \) field at \( R=R_0/3 \) will 
be zero because \( \rho = \rho(R) \). Therefore \( \ddot{\mathbf{g}} \) 
at \( R=R_0/3 \) is due only to the matter inside 
the radius \( R=R_0/3 \). The mass of this sphere 
of radius \( R_0/3 \) is

\[ M_{R=R_0/3} = \frac{4\pi}{3} \rho_0 \left( \frac{R_0^3}{3} - \frac{\alpha R_0^4}{4} \right) = \frac{4\pi}{3} \rho_0 \left( \frac{R_0^3}{3} - \frac{\alpha R_0^4}{4} \right) = \frac{4\pi}{3} \rho_0 \left( \frac{R_0^3}{3} \right) - \frac{\alpha R_0^4}{4} \]

\[ \Rightarrow \ddot{\mathbf{g}} = \frac{\rho_0}{81} \left( \frac{R_0^3}{3} - \frac{\alpha R_0^4}{4} \right) \hat{r} \]