When a mass $m_1$ is attached to the spring, the equilibrium position is $y_1 = 12.50$ cm below the support. The spring constant is $215$ N/m. Now a mass $m_2$ is attached and the system released from rest at $y_1$. (All coordinates are measured to the same point on $m_1$).

(a) Calculate the period of the resulting oscillations.

(b) Calculate the maximum downward displacement measured from the support.

(c) Calculate the energy in the oscillations only.

\[ T = 2\pi \sqrt{\frac{m_1 + m_2}{k}} = 0.868 \text{ s} \]

For $m_1$ only in equilibrium:
\[ m_1 g = k d \quad \Rightarrow \quad d = \frac{m_1 g}{k} \]

For $m_1 + m_2$ in equilibrium:
\[ (m_1 + m_2) g = k (d + s) \]
\[ m_1 g + m_2 g = k d + k s \]
\[ s = \frac{m_2 g}{k} \]

This is the amplitude of the system once it is initiated.

\[ y_2 = y_1 + 2s = y_1 + \frac{2m_2 g}{k} = 33.9 \text{ cm} \]
If there were no oscillations, the system would be in equilibrium:

\[ E_0 = \frac{1}{2}k(s+d)^2 \]

The energy of the oscillating system is equal to the potential energy at the top of its motion:

\[ E_1 = \frac{1}{2}kd^2 + (m_1 + m_2)gs \]

\[ \Delta E = E_1 - E_0 = \frac{1}{2}kd^2 + (m_1 + m_2)gs - \frac{1}{2}k(s+d)^2 \]

\[ = \frac{1}{2}kd^2 + (m_1 + m_2)gs - \frac{1}{2}ks^2 - ksd - \frac{1}{2}kd^2 \]

\[ = (m_1 + m_2) gs \cdot \frac{m_2 g}{k} - k s \left( \frac{1}{2} s + d \right) \]

\[ = (m_1 + m_2) \frac{m_2 g^2}{k} - k \left( \frac{1}{2} m_1 + \frac{m_2 g}{k} \right) \]

\[ = \frac{m_2 g^2}{k} \left( m_1 + \frac{1}{2} m_2 - \frac{1}{2} m_1 - m_1 \right) \]

\[ \Delta E = \frac{m_2 g^2}{2k} = 1.23 \text{ J} \]

Another way to get this is to note that the energy in the oscillations is the spring energy at maximum displacement:

\[ \Delta E = \frac{1}{2} ks^2 = \frac{1}{2} k \cdot \frac{m_2 g^2}{k^2} = \frac{m_2 g^2}{2k} \]

or that it is the maximum kinetic energy:

\[ y(t) = s \cdot \cos(\omega t) \]

\[ v(t) = -s \omega \sin(\omega t) \]

\[ v_{\text{max}} = s \omega \]

\[ \Delta E = \frac{1}{2} (m_1 + m_2) (s \omega)^2 = \frac{1}{2} (m_1 + m_2) s^2 \omega^2 = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) \frac{k}{m_1 + m_2} \frac{m_2 g^2}{k^2} \]

\[ = \frac{m_2 g^2}{2k} \]

\[ -U_g = 0 \]