A massless beam with a mass m on its end is pivoted at its left end and supported by two Hookes' law springs as shown. Initially spring 1 is extended from equilibrium and spring 2 is compressed.

(a) For a small amplitude of oscillation calculate the angular frequency of oscillation.

(b) Now include the effects of the beam by giving the beam a mass M and a moment of inertia of \( \frac{1}{3} ML^2 \), for rotation about the pivot. Calculate the new angular frequency of oscillation.

\[
I = ml^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{I}}
\]

for small displacements \( \tan \theta \approx \theta \)

for spring \( \# 1 \) \( \tan \theta = \theta = \frac{y_1}{L/2} \Rightarrow y_1 \), pivot

where \( y_1 = \text{dist displaced from equilibrium} \)

So, \( y_1 = \frac{L\theta}{2} \). Similarly for spring \( \# 2 \),

\( \tan \theta = \theta = \frac{y_2}{L} \Rightarrow y_2 = \frac{L\theta}{2} \)

Torque due to \( \# 1 \):

\[
F_1 = -k_1 y_1 = -k_1 \frac{L\theta}{2}
\]

So, \( \tau_1 = F_1 \cdot \frac{L}{2} = -\frac{k_1 L^2 \theta}{4} \)

Similarly for \( \# 2 \):

\[
\tau_2 = F_2 \cdot L = -k_2 L^2 \theta \quad \Rightarrow \quad \tau = I \ddot{\theta} = ml^2 \frac{d^2 \theta}{dt^2}
\]

\[
-\frac{k_1 L^2 \theta}{4} - k_2 L^2 \theta = ml^2 \frac{d^2 \theta}{dt^2} \quad \Rightarrow \quad \frac{d^2 \theta}{dt^2} + \left( \frac{k_1/4 + k_2}{m} \right) \theta = 0
\]

So, \( \omega = \sqrt{\frac{k_1/4 + k_2}{m}} \)

(b) done exactly the same way only \( I = \frac{1}{3} ML^2 + ml^2 \)

Answer is: \( \omega = \sqrt{\frac{k_1/4 + k_2}{M^2/3 + m}} \)