An organ pipe is open at both ends. By experiment, resonances are found at 655 Hz, 1048 Hz, 1703 Hz and 2227 Hz as well as others. Take the speed of sound as 330 m/s.

(a) What is the largest value of the fundamental frequency allowed by the data?

(b) If there is a pressure node at each open end of the pipe, how many pressure nodes are there between the ends for the frequency 1703 Hz?

(c) How long is the pipe?

[Solution]:

\[ L = \frac{n}{2} \lambda; \quad \lambda_n = \frac{2L}{n}; \quad \therefore f_n = \frac{v}{\lambda_n} = n \cdot \frac{v}{2L} = n f_1 \propto n \quad (n = 1, 2, \ldots) \]

Note (By reduction):

\[
\frac{1048}{655} = \frac{8}{5}, \quad \frac{1703}{655} = \frac{13}{5}, \quad \frac{2227}{655} = \frac{17}{5},
\]

Let: \( f_1 = \frac{655}{5} = \frac{1048}{8} = \frac{1703}{13} = \frac{2227}{17} \)

\[ \therefore 655 = 5f_1 = f_5 \]

\[ \begin{align*}
1048 &= 8f_1 = f_8 \\
1703 &= 13f_1 = f_{13} \\
2227 &= 17f_1 = f_{17}
\end{align*} \]

[or by SUBTRACTION:

\[
\begin{bmatrix}
655 \\
1048 \\
1703 \\
2227
\end{bmatrix}
\begin{bmatrix}
f_1 \\
f_4 \\
f_7 \\
f_{10}
\end{bmatrix}
\]

And this is the largest one; otherwise we cannot get integer n's for all f's. \[ f_1 = 131 \text{ (Hz)} \]

(b) Now \( n = 13 \): \( \therefore \) Nodes between the ends = \( n-1 = 12 \)

(c) \( f_1 = \frac{v}{L} \)

\[ \therefore L = \frac{2f_1}{v} = \frac{330}{2 \times 131} \approx 1.26 \text{ (m)} \]