While singing in the shower, we notice that the system is resonant at certain frequencies. Consider only end walls that are 8.00 ft apart (i.e., ignore effects due to side walls, ceiling, and floor).

(a) Calculate the first four frequencies at which resonant standing waves would occur between these walls. Assume the air is at 20.0°C. (\(v_{\text{sound}} = 1128.6 \text{ ft/s}\)).

\[
V = 1128.6 \text{ ft/s}
\]

Allowable wavelengths are given by the relationship:

\[
\lambda = \frac{2l}{n} \quad (8 \text{ ft}) \quad \text{and} \quad n+1 = \text{total number of nodes and} \ n = 1, 2, 3, \ldots
\]

Also, \(f = \frac{V}{\lambda} \), \(n = 1 \), \(\lambda_1 = \frac{2 \times 8}{1} = 16 \text{ ft} \);
\(n = 2 \), \(\lambda_2 = \frac{2 \times 8}{2} = 8 \text{ ft} \);
\(n = 4 \), \(\lambda_4 = \frac{2 \times 8}{4} = 4 \text{ ft} \), \(f_0 = \frac{V}{\lambda_0} = \frac{1128.6}{16} = 70.5 \text{ s}^{-1} \)

\[
f_1 = \frac{1128.6}{8} = 141.1 \text{ s}^{-1}
\]

\[
f_2 = \frac{1128.6}{16/2} = 212.5 \text{ s}^{-1}
\]

\[
f_3 = \frac{1128.6}{4} = 282 \text{ s}^{-1}
\]

(b) \(V = \sqrt{\frac{\rho P}{\mu}} \quad \text{so} \quad V \propto \frac{1}{\sqrt{\rho}} \quad V_0 \propto \frac{1}{\sqrt{\rho_0}} \quad V_f \propto \frac{1}{\sqrt{\rho_f}} \quad \frac{V_0}{V_f} = \frac{1}{\sqrt{\frac{\rho_f}{\rho_0}}} \)

Now: \(\rho_f = \rho_0 (1 - 0.025) \quad \rho_f = 0.975 \rho_0 \quad V_f = V_0 \sqrt{\frac{\rho_0}{0.975 \rho_0}} = V_0 \frac{1}{\sqrt{0.975}} \)

\[V_0 = 1128.6 \text{ ft/s} \quad \text{so} \quad V_f = \frac{1128.6}{\sqrt{0.975}} = 1143 \text{ ft/s} \]

From part (a): \(f_0 = \frac{V}{\lambda_0} = \frac{1143}{16} = 71.4 \text{ s}^{-1} \)