

EXAM 2

1

Name: _____

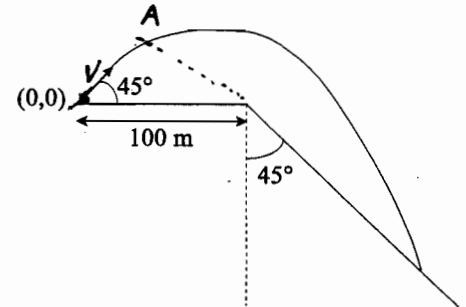
uid: u _____

Discussion TA (circle): Aaron Yuan Xiao

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!

Use the conversion constants and data given on the front page.

A projectile is launched from the point $(x = 0, y = 0)$ at an angle of 45.0° and a speed of 50.0 m/s. The ground is level for 100 m and then drops at a 45.0° slope.



- (a) Find the maximum height of the projectile.
- (b) Find the speed of the projectile 3.00 s after launch.
- (c) Find the x and y coordinates of the point of impact of the projectile.
- (d) Find the speed of the projectile at the time of impact.
- (e) Find the time of flight of the projectile.

Sol: (a) $\frac{v \sin \theta + 0}{2} \cdot \frac{v \sin \theta}{g} = \frac{v^2 \sin^2 \theta}{2g} = \frac{50^2 \times \sin^2 45^\circ}{2 \times 9.8} = 63.8 \text{ m}$

(b) $V_x = v \cos \theta$
 $V_y = v \sin \theta - gt$
 $v = \sqrt{V_x^2 + V_y^2} = \sqrt{(50 \times \cos 45^\circ)^2 + (50 \times \sin 45^\circ - 9.8 \times 3)^2} = 35.8 \text{ m/s}$

(c) The trajectory of the projectile is

$$\begin{cases} x = v \cos \theta t & (*) \\ y = v \sin \theta t - \frac{1}{2} g t^2 \end{cases} \Rightarrow y = \tan \theta \cdot x - \frac{g}{2v^2 \cos^2 \theta} x^2 = x - \frac{g}{v^2} x^2 \quad (1)$$

$$= x - 3.92 \times 10^{-3} x^2$$

The slope can be described by $y = -(x - 100) = -x + 100 \quad (2)$
 combine eq (1) and eq (2), we get

$x_1 = 454 \text{ m}, x_1 = -354 \text{ m}$
 $x_2 = 56.1 \text{ m}, y_2 = +43.9 \text{ m}$
 x_2 is the point A on the diagram. so it is not the point we want. Only (x_1, y_1) is reasonable.

(d) From eq (*), we get $t = \frac{x}{v \cos \theta} = \frac{454}{50 \times \cos 45^\circ} = 12.8 \text{ s}$

$v = \sqrt{(v \cos \theta)^2 + (v \sin \theta - gt)^2} = \sqrt{(50 \times \frac{\sqrt{2}}{2})^2 + (50 \times \frac{\sqrt{2}}{2} - 9.8 \times 12.8)^2} = 97.8 \text{ m/s}$

(e) we have solved it in d). $t = 12.8 \text{ s}$

EXAM 2

2

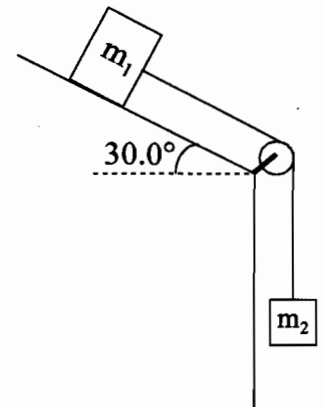
Name: _____

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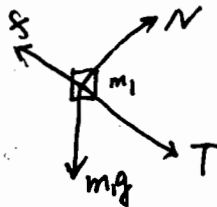
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Two blocks, $m_1 = 200$ kg and $m_2 = 100$ kg, are connected by a lightweight cord over a lightweight frictionless pulley as shown in the drawing. The coefficient of kinetic friction between the block and the incline is 0.300.



- Draw free body diagrams for each block.
- Find the acceleration of the blocks.
- Find the tension of the cord.
- Find the minimum coefficient of static friction between the block (m_1) and the incline such that the blocks do not move.

Sol: a):



$$\begin{cases} T - \mu m_1 g \cos \theta + m_1 g \sin \theta = m_1 a & \textcircled{1} \\ m_2 g - T = m_2 a & \textcircled{2} \end{cases}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow m_2 g - \mu m_1 g \cos \theta + m_1 g \sin \theta = (m_1 + m_2) a$$

$$\Rightarrow a = \frac{m_2 g - \mu m_1 g \cos \theta + m_1 g \sin \theta}{m_1 + m_2} \quad (*) = \frac{100 \times 9.8 - 0.3 \times 200 \times 9.8 \times \cos 30^\circ + 200 \times 9.8 \times \sin 30^\circ}{200 + 100}$$

$$= 4.84 \text{ m/s}^2$$

$$\textcircled{c}) \quad T = m_2 g - m_2 a = 100 \times 9.8 - 100 \times 4.84 = 496 \text{ N}$$

d) It means that the eq. (*) for a can be rewritten as

$$0 = a = \frac{m_2 g - \mu m_1 g \cos \theta + m_1 g \sin \theta}{m_1 + m_2}$$

$$\Rightarrow \mu = \frac{m_2 g + m_1 g \sin \theta}{m_1 g \cos \theta} = \frac{100 + 200 \times \sin 30^\circ}{200 \times \frac{\sqrt{3}}{2}} = 1.15$$

EXAM 2

3

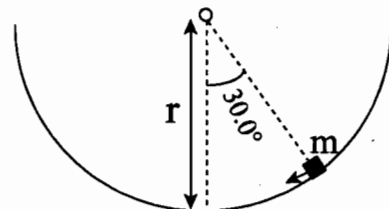
Name: Solution

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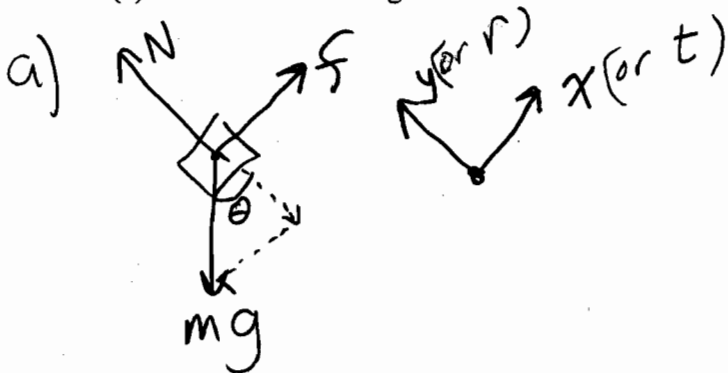
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REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A block of mass $m = 2.00$ kg slides inside a semicircular surface of radius $r = 0.500$ m as in the figure. When the block is located at an angle of 30.0° from the vertical, its speed is 0.500 m/s and is sliding downward. The coefficient of kinetic friction between the block and the surface is 0.250 .



- Draw a free body diagram for the block at that point.
- Calculate the centripetal acceleration of the block at that point.
- Calculate the normal force on the block at that point.
- Calculate the tangential acceleration of the block at that point.
- What is the magnitude of the net force on the block at that point?



e) $|\vec{a}| = \sqrt{a_t^2 + a_c^2}$
 $= 2.697 \text{ m/s}^2$
 $|\vec{F}| = m|\vec{a}| = 5.39 \text{ N}$

b) $a_c = \frac{v^2}{r} = \frac{(0.500 \text{ m/s})^2}{0.500 \text{ m}} = 0.500 \text{ m/s}^2$

c) $\sum F_y = N - mg \cos \theta = \frac{mv^2}{r}$
 $\Rightarrow N = mg \cos \theta + \frac{mv^2}{r}$
 $= 16.97 \text{ N} + 1.00 \text{ N} = 18.0 \text{ N}$

d) $\sum F_x = f - mg \sin \theta = -ma_t \Rightarrow a_t = g \sin \theta - \frac{\mu N}{m} = 2.65 \text{ m/s}^2$

$\begin{matrix} 4.90 & 2.50 & 18.0 \\ \uparrow & \uparrow & \uparrow \\ g \sin \theta & - \frac{\mu N}{m} & \end{matrix}$
 $\searrow 2.00$