Physics 2210
Spring Semester 2008
Anthony Pantziris

FINAL EXAM

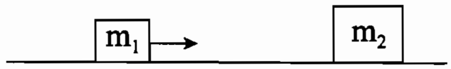
1

Name: ______________________ unid: u _ _ _ _ _ _ _

Discussion TA (circle): Aaron Yuan Xiao

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A block of mass $m_1 = 2.00$ kg collides head-on with a mass $m_2 = 5\ m_1$ initially at rest on a horizontal surface. The mass m_1 strikes m_2 with a speed of 4.00 m/s.



5' (a) Assuming the collision is perfectly inelastic and the objects stick together, calculate their speed right after the collision.
5' (b) Calculate the mechanical energy lost in this collision.
5' (c) If the coefficient of friction between the blocks and the surface is $\mu_k = 0.250$, calculate how long it would take for the masses to stop after they stick together.
5' (d) Now assume the collision is elastic and calculate the speeds of the two masses immediately after the collision.

(a) $m_1 V_1 = (m_1 + 5m_1) V_f$

$$V_f = \frac{m_1 V_1}{6 m_1} = \frac{V_1}{6} = 0.667\ m/s$$

(b) $\Delta E = E_i - E_f = \frac{1}{2} m_1 V_1^2 - \frac{1}{2}(m_1+m_2)\left(\frac{V_1}{6}\right)^2$

$$= \frac{1}{2} m_1 V_1^2 - \frac{1}{2} 6 m_1 \cdot \frac{V_1^2}{36}$$

$$= \frac{5}{12} m_1 V_1^2$$

$$= \frac{5}{12} \times 2 \times 4^2 = 13.3\ J$$

(c) $f = \mu_k \cdot N = \mu_k (m_1 + m_2) g = (m_1 + m_2) a$

$$a = \mu_k g = 2.45\ m/s^2$$

$$V_f - V_i = -at$$

$$0 - 0.667 = -2.45 t$$

$$t = \frac{0.667}{2.45} = 0.27 s$$

(d) $\begin{cases} m_1 V_1 = m_1 V_{1f} + m_2 V_{2f} \quad \cdots ① \\ \frac{1}{2} m_1 V_1^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 \cdots ② \end{cases}$

from equation ① $\Rightarrow V_{1f} = V_1 - 5V_{2f} \cdots ③$

put equation ③ back to equation ②

$$V_1^2 = (V_1 - 5V_{2f})^2 + 5V_{2f}^2$$

$$30 V_{2f}^2 - 10 V_{2f} V_1 = 0$$

$\begin{cases} V_{2f} = \frac{1}{3} V_1 \\ V_{2f} = 0 \times \text{ no collision} \end{cases}$

so: $V_{1f} = -\frac{2}{3} V_1 \qquad |V_{1f}| = 2.67\ m/s$

$V_{2f} = \frac{1}{3} V_1 \qquad |V_{2f}| = 1.33\ m/s$

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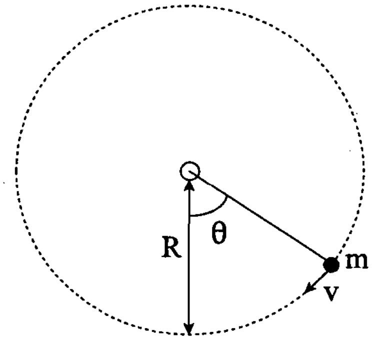
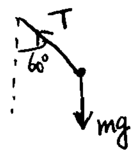
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A particle of mass m = 3.00 kg attached to the end of a string swings in a *vertical* circle of radius R = 2.50 m. At the instant when θ = 60.0°, the tension in the string is 80.0 N.



(a) Draw a free body diagram for the particle.
(b) Calculate the centripetal acceleration and the speed of the particle at that instant.
(c) Calculate the magnitude of the tangential acceleration of the particle at that instant.
(d) Calculate the magnitude of the resultant force on the particle at that instant.
(e) Calculate the kinetic energy of the particle at the bottom of the circle.

Sol: a)



b) $\Sigma F_t = m a_c = m\frac{v^2}{R} = T - mg\cos\theta$

$a_c = \frac{T - mg\cos\theta}{m} = \frac{80 - 3\times 9.8\times\frac{1}{2}}{3} = 21.7667$

$\boxed{a_c = 21.8\ \frac{m}{s^2}}$ $\quad V = \sqrt{a_c R} = \sqrt{21.7667\times 2.5} = 7.3767$

$\boxed{V = 7.38\ \frac{m}{s}}$

c) $\Sigma F_t = m a_t = mg\sin\theta \qquad a_t = g\sin\theta = 9.8\times 0.866 = 8.487$

$\boxed{a_t = 8.49\ m/s^2}$

d) $|\vec{F}| = m|a| = m\sqrt{a_t^2 + a_c^2} = 3\cdot\sqrt{(8.487)^2 + (21.766)^2} = 70.088$

$\boxed{|\vec{F}| = 70.1 N}$

e) $E_\theta = E_{bottom} \qquad \frac{1}{2}mv^2 + mgh = K_B = m\left(\frac{V^2}{2} + gR(1-\cos\theta)\right)$

$K_B = 3\left(\frac{7.37^2}{2} + 9.8\times 2.5\times\frac{1}{2}\right) = 118.37$

$\boxed{K_B = 118\ J}$

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REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A planet in another solar system has radius $R = 6.34 \times 10^6$ m. It has a small moon of mass $m = 2.50 \times 10^{19}$ kg which is in a circular orbit around the planet at a distance of $h = 3.20 \times 10^8$ m above its surface with a period of 15.6 (Earth) days.

(a) Find the mass of the planet.
(b) Calculate the gravitational acceleration at the surface of the planet.
(c) Calculate the kinetic energy of the moon.
(d) Determine the total energy of the moon.
(e) Calculate the escape speed from the surface of the planet.

sol: a) $\frac{G M_p M_m}{(R+h)^2} = \frac{M_m V^2}{(R+h)}$, $V = \frac{2\pi(R+h)}{T}$, $T = 15.6 \times 24 \times 3600 = 1.348 \times 10^6$ s

$$\Rightarrow \frac{G M_p}{R+h} = \frac{[2\pi(R+h)]^2}{T^2}$$

$$\Rightarrow M_p = \frac{4\pi^2 (R+h)^3}{T^2 G} = \frac{4 \times 3.14^2 \times (6.34 \times 10^6 + 3.20 \times 10^8)^3}{(6.67 \times 10^{-11}) \times (1.348 \times 10^6)^2} = 1.13 \times 10^{25} \text{ kg}$$

b) $g = \frac{G M_p}{R^2} = 18.8 \text{ m/s}^2$

c) $E_K = \frac{1}{2} M_m V^2 = \frac{1}{2} m_m \times \left(\frac{2\pi(R+h)}{T}\right)^2 = 2.89 \times 10^{25}$ J

d) $E_{tot} = \frac{1}{2} m_m V^2 - \frac{G M_p m_m}{R+h} = -2.89 \times 10^{25}$ J

e) $V_{esc} = \sqrt{\frac{2 G M_p}{R}} = \sqrt{\frac{2 \times 6.673 \times 10^{-11} \times 1.13 \times 10^{25}}{6.34 \times 10^6}} = 1.54 \times 10^4$ m/s

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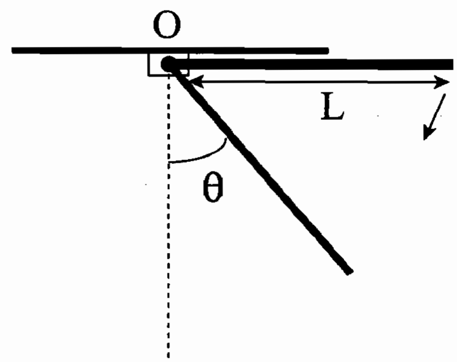
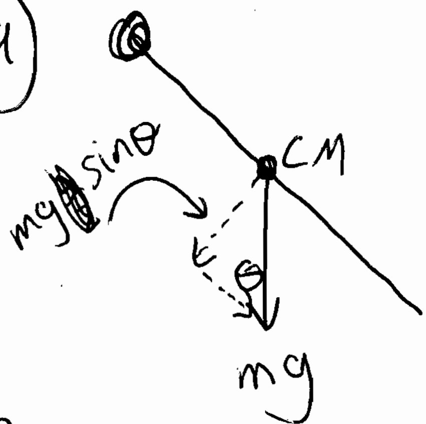
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end. The rod is released from rest in the horizontal position. Put your answers in terms of L, M, g and θ.



(a) Calculate the torque of the weight of the rod about the axis of rotation O when the rod makes an angle θ with the vertical.
(b) Calculate the angular acceleration of the rod at that point.
(c) Find the kinetic energy of the rod at that point.
(d) Find the angular speed of the rod at that point.
(e) Determine the speed of the center of mass of the rod at that point.

a)



$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = mg\frac{L}{2}\sin\theta$

rotation around center

b) $\tau = I\alpha \Rightarrow mg\frac{L}{2}\sin\theta = \left(\frac{1}{3}ML^2\right)\alpha \Rightarrow \alpha = \frac{3}{2}\frac{g\sin\theta}{L}$

c) $\cancel{KE_i} + PE_i = KE_f + \cancel{PE_f}$ choose $PE_f = 0$

$mg\frac{L}{2}\cos\theta \left(= \frac{1}{2}I\omega^2\right) = KE$

$KE = mg\frac{L}{2}\cos\theta$ (answers in terms of L, M, g, θ

d) $\frac{1}{2}I\omega^2 = mg\frac{L}{2}\cos\theta \Rightarrow \omega = \sqrt{\frac{3g\cos\theta}{L}}$

e) $v = r\omega = \frac{L}{2}\omega = \frac{1}{2}\sqrt{3gL\cos\theta}$

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REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
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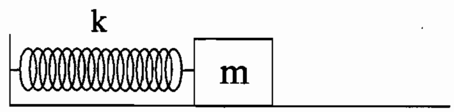
The block shown is released from rest on a rough surface when the spring is stretched a distance of d = 10.0 cm. Given: m = 0.500 kg, k = 50.0 Nm and $\mu_k = 0.250$.

(a) Calculate the speed of the block when it first passes through this position for which the spring is unstretched.
(b) Calculate the acceleration of the block when it first passes through the position for which the spring is unstretched.
(c) Calculate the maximum compression of the spring after it first passes through the position for which the spring is unstretched.



a) unstretched = equilibrium ⇒ KE is max, $PE_{elastic} = 0$

$E_i = E_f$ non-conservative work

$\frac{1}{2}kd^2 - \mu_k m g d = \frac{1}{2} m v_B^2 \Rightarrow v_B = .714$ m/s

b) at equilibrium there is no spring force, but still have friction

$\Sigma F = \mu_k m g = ma \Rightarrow a = \mu_k g = 2.45$ m/s² (magnitude)

c) $E_i = E_f$

$\frac{1}{2} m v_B^2 - \mu_k m g x = \frac{1}{2} k x^2$

$\Rightarrow \left(\frac{1}{2}k\right)x^2 + (\mu_k m g)x - \frac{1}{2} m v_B^2 = 0$ quadratic

$x = 5.10E-2$ m (negative solution ignored) since before equilibrium