A. A car (mass $m = 1400 \text{ kg}$) is driving on a frictionless, circular track with radius of curvature $R = 100 \text{ m}$, as shown. The car’s path is determined by an electric force between a charge $q$ fixed to the car’s center of mass and a charge $Q = +0.5 \text{ C}$ at the center of the path. Find the magnitude of charge $q$ necessary to keep the car on the circular track when its speed is $v = 75 \text{ m/s}$.

$$q = \frac{0.175 \text{ C}}{2}$$

$$\sum F_{\text{radial}} = \frac{KqQ}{r^2} = ma_c = \frac{mv^2}{r}$$

$$\Rightarrow \frac{mv^2}{KQ} = 0.175 \text{ C} = q$$

Since the force must pull into the circle, we get $q = -0.175 \text{ C}$ and then $|q| = 0.175 \text{ C}$

B. Calculate electric field $\vec{E}$ at the origin in the presence of two charges $q_1 = +0.03 \text{ C}$ and $q_2 = -0.01 \text{ C}$, at $(x_1,y_1) = (0,2)$ and $(x_2,y_2) = (2,0)$, respectively, where coordinate distances are in units of meters. Write your answer in terms of unit vectors $\hat{i}$ and $\hat{j}$.

$$\vec{E} = 2.39 \times 10^{-7} \hat{i} - 1.39 \times 10^{-7} \hat{j}$$

$$\vec{E} = \frac{Kq}{r^2} \Rightarrow E_1 = \frac{Kq_1}{r_1^2}, \quad E_2 = \frac{Kq_2}{r_2^2}$$

$$= 3.38 \times 10^{-7} \text{ N/C} = -1.00 \times 10^{-7} \text{ N/C}$$

$\tan \theta = \frac{E_2}{E_1} \Rightarrow \theta = 45^\circ$

$$r_1 = \sqrt{2^2 + 2^2} = 2\sqrt{2} \text{ m}$$

$$r_2 = 3 \text{ m}$$

Then we use superposition and add them together.

$E_{\text{total}} = E_1 \cos \theta + E_2 = 2.39 \times 10^{-7} \hat{i}$

For $y$, the $E_1$ gives an $E$ field in the $-y$ direction and $E_2$ in the positive $y$ so.

$$E_y = -E_1 \sin \theta - E_2 = -1.39 \times 10^{-7} \hat{j}$$
A straight segment of wire, length \( L \), has a uniform charge per unit length, \( \lambda \).

A. Derive the strength of the electric field, \( E \), measured at a point on the wire's axis at a distance \( D \) away from the origin, as shown:

\[
|\lambda| = \frac{dQ}{dx}, \quad \left| dE_x \right| = k \frac{|dQ|}{r^2}, \quad r = D - x
\]

\[
|E_x| = \int_0^L k \frac{|\lambda|}{(L-x)^2} dx \quad (x \neq L), \quad b = -1, \quad u = 0
\]

\[
|E_x| = k |\lambda| \left[ \frac{\lambda}{(a+b)x} \right]_x=0
\]

\[
|E_x| = k |\lambda| \left[ \frac{1}{D-x} \right]_{x=0} = \left| k |\lambda| \left[ \frac{1}{D-L} - \frac{1}{D} \right] \right|
\]

Alternative,

\[
E_x = k |\lambda| \left[ \frac{1}{D-x} \right]_{x=0} = \left| k |\lambda| \left[ \frac{1}{D-L} - \frac{1}{D} \right] \right|
\]

B. What is the electric potential \( V \) at that same point \( P \) (relative to a point at infinity)?

\[
V_p = -\int_\infty^P E_x \cdot d\hat{x}
\]

Choose path along \( x = \alpha \) for \( (x \geq D) \):

\[
d\hat{x} = dx \hat{x}, \quad E(x) = k |\lambda| \left[ \frac{1}{x-L} - \frac{1}{x} \right] \quad \text{(sign determined)}
\]

\[
V_p = -\int_\infty^D k |\lambda| \left[ \frac{1}{x-L} - \frac{1}{x} \right] dx = k |\lambda| \left[ \ln \left( \frac{x_L}{x} \right) \right]_{x=\infty}^x
\]

\[
V_p = k |\lambda| \left[ \ln \left( \frac{D-L}{D} \right) - \lim_{x \to \infty} \ln \left( \frac{x-L}{x} \right) \right]
\]

\[
V_p = k |\lambda| \ln \left( \frac{D-L}{D} \right)
\]

Again, sign of \( V_p \) is determined only by \( \lambda \) as expected.

C. A jumble of such wire segments creates an electric potential at point \((x,y,z)\) such that \( V = 3x - 2/z \) (coordinates in units of meters gives \( V \) in units of Volts—relative to a point at \( \infty \)). Find the electric field \( E \) at that point in terms of unit vectors \( \hat{i}, \hat{j}, \hat{k} \).

\[
E = -3\hat{z} - \frac{2}{z^2} \hat{k}
\]

\[
E = -\nabla V = -\left( \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \left( 3x - \frac{2}{z} \right) = -3\hat{z} - 0\hat{j} - \frac{2}{z^2} \hat{k}
\]
Show all work!! Report all numbers to three (3) significant figures.

An insulating sphere of radius \( a \) lies centered within a spherical conducting shell (inner radius and outer radius \( c \)). The charge per unit volume in the insulating sphere is \( +p \), a constant, while the total charge on the conducting shell is zero.

A. Find the electric field strength \( E \) at radius \( r \) inside the insulating sphere \( (r < a) \).

\[
E = \frac{\rho r}{3\varepsilon_0}
\]

B. Find the surface charge density \( \sigma_c \) on the outer surface of the conductor.

\[
\sigma_{\text{out}} = \frac{q_{\text{out}}}{4\pi c^2} = \frac{q_{\text{in}} + q_{\text{out}}}{4\pi c^2} = \frac{q_{\text{in}}}{4\pi c^2} = \frac{1}{4\pi c^2} \left( \frac{4}{3} \pi a^3 \right) \rho = \frac{\rho a^3}{3c^2}
\]

C. Find the electric potential \( V \) on the inner surface of the conductor (relative to a point at infinite distance).

\[
\Delta V = -\int_a^b E \cdot ds \quad \Delta V_{c \to b} = 0 \quad V(c) = V(b)
\]

\[
\Delta V = -\int_{\infty}^c \frac{\rho a^3}{3\varepsilon_0 r^2} dr = -\frac{\rho a^3}{3\varepsilon_0} \int_{\infty}^c \frac{1}{r^2} dr = \frac{\rho a^3}{3\varepsilon_0 c}
\]

D. If the potential (relative to infinity) of the outer surface of the conductor is \( V_c = 110 \text{ V} \), what is the speed of an electron, \( v \), which hits that surface of it, that has been released from rest at a large (infinite) distance to fall onto the conductor?

\[
\Delta U = -\Delta k \quad u_f - u_i = k_f - k_i \quad k_f = -U_f
\]

\[
u = \sqrt{\frac{2eV_i}{m_e}} \quad V = 6.22 \times 10^8 \text{ m/s}
\]