A 12 volt battery is connected to two resistors ($R_1 = 5.00 \, \Omega$ and $R_2 = 7.00 \, \Omega$) and an inductor ($L = 3.00 \, \text{mH}$). Switch $S$ has been open for a long time and is closed at time $t = 0 \, \text{s}$.

**A.** [10 pts.] What is the magnitude of the current $I_1$ immediately after the switch is closed?

**B.** [15 pts.] What is the magnitude of $dI_1/dt$ immediately after the switch is closed?

**C.** [10 pts.] What is the magnitude of the current $I_L$ after the switch has been closed for a long time?

**D.** [10 pts.] After the switch has been closed for a long time, it is opened. The current $I_L$ now decays exponentially as a function of time. What is the value of the time constant of this decay?

**SHOW ALL WORK!**

![Circuit Diagram]

\[ E = I_1 R_1 + I_2 R_2 \]
\[ V_c = E - I_1 R_1 \]

**A.**

\[ V_c = E \Rightarrow \frac{E}{R_1} + \frac{V_c}{R_2} = \frac{E}{R_1} \]

\[ I_1 = \frac{E}{R_1} \]

\[ I_2 = \frac{E}{R_2} \]

\[ E = I_1 (R_1 + R_2) \]

\[ \frac{dI_1}{dt} = \frac{12 \, \text{V}}{3 \times 10^{-3} \, \text{H}} \left(1 - \frac{5 \, \Omega}{12 \, \Omega}\right) = 2.33 \times 10^{-2} \, \text{A/s} \]

\[ \frac{dI_1}{dt} = \frac{12 \, \text{V}}{5 \times 10^{-3} \, \text{H}} = 2.4 \, \text{A} \]

**B.**

\[ \frac{E}{R_1} \]

**C.**

\[ \frac{L}{R_2} = \frac{3 \times 10^{-3} \, \text{H}}{7 \, \Omega} = 4.29 \times 10^{-4} \, \text{s} \]

**D.**
SHOW ALL WORK!

The switch in the figure has been in position a for a long time, completely charging the capacitor. At time $t = 0$ s, the switch is moved to position b. Let $I(t)$ be the current through the inductor. Let $e = 9.00$ V, $L = 8.00$ mH, and $C = 20$ μF.

A. [13 pts.] What is the maximum magnitude of $I$?

B. [12 pts.] Find the earliest time after $t = 0$ s that the magnitude of the current $I$ is at its maximum value. It may help to sketch the current as a function of time.

\[
\text{Energy} = \frac{1}{2} CV_{\text{max}}^2 = \frac{1}{2} C e^2 = \frac{1}{2} LI_{\text{max}}^2
\]

\[
I_{\text{max}} = \frac{e}{\sqrt{LC}} = 0.45 \text{ A}
\]

Alternatively,

\[
I(t) = I_{\text{max}} \cdot \sin (\omega t + \phi)
\]

\[
I_{\text{max}} = I_{\text{max}} \sin (\omega t)
\]

$1 = \sin \omega t$

First possible time when

\[
\omega t = \frac{\pi}{2}
\]

\[
\Rightarrow e = 6.28 \times 10^{-4} \text{ s}
\]
SHOW ALL WORK!

The next three questions refer to the RLC circuit shown in the figure. Let $e = 200 \sin(\omega t)$ volts, $L = 5.00 \text{ mH}$, $C = 10.0 \text{ \mu F}$, and $R = 30.0 \text{ \Omega}$.

A. [10 pts.] At what frequency, in Hertz, is $I_{\text{rms}}$ maximum?

B. [10 pts.] What is the maximum possible magnitude of $I_{\text{rms}}$?

C. [10 pts.] Is the current through the capacitor leading, lagging, or in phase with the voltage across the resistor? Explain how you know this.

\[ X_L = X_C \quad \Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}} = \frac{712}{\text{Hz}} \]

\[ I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{E_{\text{max}}}{Z} = \frac{1}{\sqrt{2}} \cdot \frac{E_{\text{max}}}{\sqrt{R^2 + (X_L - X_C)^2}} \]

\[ = \frac{1}{\sqrt{2}} \cdot \frac{E_{\text{max}}}{R} = \left(\frac{200V}{30\Omega}\right) \cdot \frac{1}{\sqrt{2}} = 4.78 A \]

1. There is only ONE current in the loop (flows through every device in the circuit)
2. The circuit is driven (by $e$) at resonant frequency.

\[ E(t) = R \cdot I(t) = V_e(t) \]

\[ \therefore I_C \text{ is IN Phase with } V_e \]