We now apply these principles to familiar optical devices. We will consider five examples: the eye, a camera, a magnifying glass, a compound microscope, and a telescope.

THE EYE

Despite its obvious biological and psychological complications, the eye is extremely simple optically, consisting of a single variable focal length lens and a detector placed at the image. Schematically it is as shown in the following sketch:

![Eye Diagram]

The retina consists of a densely packed array of light sensitive cells roughly 5 microns in diameter. They are among the smallest cells in the body. When they absorb a photon (light “particle”) they produce a voltage which is propagated along the optic nerve to the visual cortex where in some only vaguely understood way they produce the phenomenon of sight. The key point is that for this to work an actual photon must be absorbed. Thus the image at the retina must be real, not virtual.

Since the distance from the lens to the detector is fixed, we must vary the focal point as the object distance changes:

\[
\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{s + s'}{ss'} \rightarrow f = \frac{ss'}{s + s'}
\]

In most cases we have \(s >> 1.8\) cm. Thus:

\[f = s' = 1.8\text{ cm}\]

The lens can be “squeezed” by muscles, reducing the radius of curvature and therefore the focal length (recall the lens maker’s equation derived earlier). In the normal (distant object) situation
this is not necessary and the muscles are completely relaxed. Unfortunately they cannot “stretch” the lens to increase the focal length. In near sighted people this relaxed position gives too short a focal point and produces an image in front of the retina, resulting in blurred vision. This is then corrected by putting another lens in front of the eye (glasses or contacts) to increase the focal length. At the other extreme there is a limit to how much the muscles can reduce the focal point, and thus a shortest object distance at which the image will be sharp. This distance is called the “near point” and for a generic human being is about 25 cm. However with age the lens becomes stiffer and harder to compress. The result is usually a near point which becomes steadily larger with advancing age. Ultimately it may reach the point where the focal length is nearly fixed. At this point people generally use bifocals – glasses with two fixed focal points, one for close objects and one for distant ones.

Finally we note that the size of the retinal cells limits the angular resolution of which the eye is capable. To see this consider the following sketch:

\[
\theta_{\text{min}} = \frac{h_{\text{min}}}{L} = \frac{5 \times 10^{-4} \text{ cm}}{1.8 \text{ cm}} \approx 3 \times 10^{-4} \text{ rad}
\]

If \( h \) is the size of the retinal cells then it is also the smallest distance on the retina which the system can detect. Hence the size of these cells limits human eye resolution to about \( 3 \times 10^{-4} \) radians. We will see later that there is another limit imposed by diffraction. In most cases they are similar.
CAMERAS

We now turn to simple cameras. They are essentially the same as the eye, but with a different detector and a fixed focal length lens which can be moved to keep the image in focus at the detector. The schematic is shown below:

As with the eye we must have a real image in order for the detector to work. There are two different detectors used in cameras – film and CCDs. Film is basically a sheet divided into “grains” of a particular size. When a grain absorbs a photon it undergoes a chemical reaction which changes it to a form which will darken or lighten when exposed to appropriate chemicals. In other words it is just like the retina except that the change in the “cells” is chemical rather than electrical. As before, it is the size of the grains that determines the angular resolution of the film.

The other type of detector, CCD’s, consists basically of little photocells which produce a voltage when they absorb a photon. These voltages are then recorded in a digital memory and form the image. Each CCD is called a “pixel”, and it is the number of pixels that determine the resolution of the detector. More about this in a bit.

Unlike the eye, the lens has a fixed focal length so that in order to keep the image at the detector the position of the lens must change. Thus cameras are focused by moving the lens. The size of the image is given by:

\[ h' = -h \frac{s'}{s} \]

\[ s' = \frac{sf}{s - f} \]

\[ |h'| = |h| \frac{f}{s - f} \]

Normally the object distance is much greater than the focal length of the lens. Hence:
For film cameras the most common size for the film is 35 mm. The most common subject is a person (~6 ft), and the most common object distance is ~8 ft. Thus we want:

\[ \frac{f}{35 \text{ mm}} = \frac{f}{6 \text{ ft}} \rightarrow f = 35 \times \frac{4}{3} \approx 50 \text{ mm} \]

For this reason most SLR cameras come with a lens of focal length ~50 mm. If you want to take more distant pictures you get a telephoto lens with longer focal length. For example a lens of \( f = 400 \text{ mm} \) would give a factor of 8 multiplication (would fill the film at 64 ft for a 6 ft person).

However, there is a second consideration, and that is the length of time the shutter has to be open to expose the film. If you want to take action shots you need short exposure times to avoid blurring due to the motion of the subject. To see how this works, consider a camera with a lens of diameter \( D \). If the intensity of the ambient light is \( I_0 \), then the amount of light entering the camera is:

\[ P = I_0 A = \frac{I_0 \pi D^2}{4} \]

The intensity at the image is then:

\[ I = \frac{P}{\text{Area}} = \frac{P}{(h')^2} = \frac{I_0 \pi D^2}{4(h')^2} \]

If there are \( n \) grains/area on the film, the power striking a grain will be:

\[ P_G = \frac{I}{n} = \frac{I_0 \pi D^2}{n4(h')^2} \]

But

\[ h'^2 = h^2 \frac{f^2}{s^2} = \gamma f'^2 \]

Hence

\[ P_G = \frac{I_0 \pi \left( \frac{D}{4 \gamma n \sqrt{f}} \right)^2}
Now it takes a certain amount of energy to activate the grain. Suppose that energy is \( U \). Then we will have to leave the shutter open for a time \( t_s \) given by:

\[
P_G t_s = U \rightarrow t_s = \frac{U}{P_G} = \frac{U4\gamma n}{I_0 \pi} \left( \frac{f}{D} \right)^2
\]

Now let

\[
\frac{4n}{\pi} = S, \quad f_# = \frac{f}{D}
\]

Then

\[
t_s = \frac{\gamma f_#^2}{I_0 S}
\]

\( I_0 \) and \( \gamma \) are fixed by the conditions under which the picture is taken. \( S \) is called the speed of the film (ASA #) and \( f_# \) is called the f-number of the lens. Comparing the results for magnification and shutter time we see that to take long range photos of action we need large \( f \) and very large \( D \). An exactly parallel discussion works for CCD cameras with pixels replacing grains.

It is interesting to ask how many pixels are required to get an acceptable picture. The result depends on the use to which it is to be put. If it is merely to be displayed, as is, on a computer screen the answer is a number of pixels equal to the number on the screen – typically \( \sim 1000 \times 2000 = 1 \) megapixel. If you want to enlarge it on the screen you will need correspondingly more pixels. If you want to print the picture and observe it the criterion is a bit different. Suppose you want to make an 8” × 10” picture. You want to view it without seeing the individual pixels. We have seen that the resolution of the eye is about \( 3 \times 10^{-4} \) radians. We can’t view it from closer than the near point of 25 cm. Hence the spacing between pixels must be less than \( 3 \times 10^{-4} \times 25 = 7.5 \times 10^{-3} \) inches. Thus we need:

\[
N = \left( \frac{10}{7.5 \times 10^{-2}} \right)^2 = 1.8 \times 10^6 \text{ pixels}
\]

On the other hand if we want to blow the picture up to 16” × 20” we need 4 times as many pixels, etc.

In both cases increased resolution is paid for by longer shutter times.
MAGNIFYING GLASS

We now consider the simple magnifying glass. It consists of a single converging lens. Since we want the image to be erect, we must have the image distance negative. This means a virtual image. The situation is then as shown in the sketch below.

What matters is the angle subtended by the image at the eye. This is the angle $\theta$ shown in the sketch above. Clearly it is given by:

$$\tan \theta = \frac{h}{s} \quad \text{since} \quad \theta_n << 1 \Rightarrow \theta_n$$

Since $\theta$ is normally small we can approximate tan by the angle. Now what we are really concerned with is the angular size with the magnifying glass compared to that without it. But without it the object can’t be closer than the near point of the eye. Hence:

$$\frac{\theta_n}{\theta_0} = \frac{h}{s} = \frac{NP}{s}$$

But

$$s = \frac{fs'}{s'-f}$$

$$\therefore \frac{\theta_n}{\theta_0} = \frac{NP(s'-f)}{sf}$$

If we take $s'$ infinite the eye will be relaxed (recall description of its operation) and hence this will be the most comfortable. Then:
If, on the other hand, we place the image at the near point we get:

\[
\frac{\theta_n}{\theta_0} \xrightarrow{s' \to \infty} \frac{NP}{f}
\]

(Note that \(s' < 0\)). In either case we would like \(f\) as small as possible. This is limited by the need to avoid distortion by making the radius of curvature large compared to the diameter of the lens. As a practical matter this limits \(f\) to greater than about 5mm.

**COMPOUND MICROSCOPE**

The arrangement is shown in the sketch below.

The system consists of two converging lenses separated by a distance \(L\). The first lens is the objective and the second is the eyepiece. The idea is to form a real image with the objective that lies just inside the focal point of the eyepiece. The eyepiece then gives an enlarged virtual image. We analyze this as follows:

\[
s'_o = \frac{sf_0}{s-f_0} = L - f_e
\]

(to put image just inside focal point of eyepiece). To accomplish this we need
\[ s = \frac{s'f_0}{s' - f_0} = \frac{(L - f_e)f_0}{L - f_e - f_0} \]

Then

\[ h_1' = -\frac{(L - f_e)(L - f_e - f_0)h}{(L - f_e)f_0} = -h \frac{(L - f_e - f_0)}{f_0} \]

\[ \theta_f = \frac{|h'|}{f_e} = \frac{h (L - f_e - f_0)}{f_0 f_e} \]

\[ \theta_0 = \frac{h}{N_p} \]

\[ \frac{\theta_f}{\theta_0} = \frac{(L - f_0 - f_e)N_p}{f_0 f_e} \]

Since this image is virtual we cannot use it to make a photograph. To do so we use only the objective lens giving a magnification which we find as follows.

\[ \frac{h'}{h} = -\frac{f_0}{s - f_0} \]

\[ s = \frac{L f_0}{L - f_0} \]

(placement image at end of microscope). Then

\[ \frac{h'}{h} = -\frac{f_0}{L f_0 - f_0} = -\frac{f_0 (L - f_0)}{L f_0 - f_0 + f_0^2} = -\frac{(L - f_0)}{f_0} \approx -\frac{L}{f_0} \]

**REFRACTING TELESCOPE**

We now consider a simple refracting telescope. The arrangement is schematically the same as for the microscope. However, now the object is at great distance compared to the focal lengths.
In this case the image will be at the focal point of the objective lens. We can then analyze the situation as follows:

\[ h' = -h \frac{f_0}{s-f_0} \approx -\frac{hf_0}{s} \]

\[ \theta_f = \frac{|h'|}{f_e} = \frac{h \cdot f_0}{s \cdot f_e} \] (relaxed eye)

\[ \theta_0 = \frac{h}{s} \]

\[ \therefore \frac{\theta_f}{\theta_0} = \frac{f_0}{f_e} \]

Again this is a virtual image and can’t be used to take a photograph. To do that we only use the objective lens. This is analyzed as follows.

\[ |h'| = h \frac{f_0}{s} \]

Hence the size of the image on the film is just proportional to the focal length of the objective lens.

**RELECTING TELESCOPE**

We now consider a reflecting telescope. Now the arrangement is as shown below.
The mirror makes a real image which is deflected to the eyepiece and lies just inside the focal point of the eyepiece. Then we analyze the situation as follows:

\[ s_0' = f_M = \frac{R_M}{2} \]

\[ |h_0'| = h \frac{f_M}{s} \]

\[ \theta_f = \frac{|h_0'|}{f_e} = \frac{hf_M}{sf_e} \]

\[ \theta_0 = \frac{h}{s} \]

\[ \theta_0 = \frac{f_M}{f_e} = \frac{R_M}{2f_e} \]

As before this image is virtual and can’t be used for a photograph. To take a photograph we used only the image produced by the mirror resulting in an image size of:

\[ |h'| = h \frac{f_M}{s} = h \frac{R_M}{2s} \]

This is again just proportional to the focal length of the mirror.

As discussed in class, reflectors are the telescope of choice for astronomical purposes. However for other uses – such as binoculars – refractors are often used.