Show all work!! Report all numbers to three (3) significant figures.

[30 pts.] Four charged particles are at the corners of a square of side \( a \) as shown in the figure below. Let \( A = 4 \), \( B = 3 \), and \( C = -2 \).

A. Determine the electric field at the location of charge \( q \) in magnitude and direction. (Use the following as necessary: \( q \), \( a \), and \( k_e \). Direction is measured counterclockwise from + \( x \)-axis)

\[
\vec{E}_q = \vec{E}_1 + \vec{E}_2 + \vec{E}_3
\]

\[
\vec{E}_1 = \frac{k_e q}{a^2} \hat{\imath}, \quad \vec{E}_2 = \frac{k_e q}{a^2} \hat{j}
\]

\[
\vec{E}_3 = \frac{k_e q}{a^2} \left\{ \left( 4 + \frac{3}{\sqrt{12}} \right) \hat{\imath} + \left( 2 + \frac{3}{\sqrt{12}} \right) \hat{j} \right\} = \left( 3.06 \hat{\imath} + 5.06 \hat{j} \right) \frac{k_e q}{a^2}
\]

\[
|\vec{E}_3| = \sqrt{(3.06)^2 + (5.06)^2} = 5.91 \frac{k_e q}{a^2} \quad \text{Ans}
\]

\[
\theta = \tan^{-1} \left( \frac{E_y}{E_x} \right) = \tan^{-1} \left( \frac{3.06}{5.06} \right) \approx 31.16^\circ \quad \text{Ans}
\]

B. Determine the total electric force exerted on \( q \) in magnitude and direction. (Use the following as necessary: \( q \), \( a \), and \( k_e \). Direction is measured counterclockwise from + \( x \)-axis)

\[
\vec{F}_q = q \vec{E}_q
\]

\[
= \frac{k_e q^2}{a^2} \left( 3.06 \hat{\imath} + 5.06 \hat{j} \right)
\]

\[
|\vec{F}_q| = \frac{k_e q^2}{a^2} 5.91 \quad \text{Ans}
\]

\[
\theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = 31.16^\circ \quad \text{Ans}
\]
Show all work!! Report all numbers to three (3) significant figures.

[35 pts.] In 1911, Ernest Rutherford and his assistants Geiger and Marsden conducted an experiment in which they scattered alpha particles (nuclei of helium atoms) from thin sheets of gold. An alpha particle, having charge \( q_a = +2e \) and mass \( m_a = 6.64 \times 10^{-27} \text{ kg} \), is a product of certain radioactive decays. The results of the experiment led Rutherford to the idea that most of an atom's mass is in a very small nucleus, with electrons in orbit around it. Assume an alpha particle, initially very far from a stationary gold nucleus, is fired with a velocity of \( v_0 = 2.35 \times 10^7 \text{ m/s} \) directly toward the nucleus (charge \( q_n = +79e \)). What is the smallest distance \( r \) between the alpha particle and the nucleus before the alpha particle reverses direction? Assume the gold nucleus remains stationary. Give answer in fm.

Conservation of Energy

\[
E_i = \frac{1}{2} m v_0^2 \quad E_f = \frac{K_e q_a q_n}{r}
\]

\[E_i = E_f \quad \Rightarrow \quad r = \frac{2 K_e q_a q_n}{m v_0^2}\]

Plug everything in

\[r = 19.8 \text{ fm}\]
Show all work!! Report all numbers to three (3) significant figures.

[35 pts.] A wire having a uniform linear charge density \( \lambda \) is bent into the shape shown in the figure below. Find the electric potential at point \( O \). Use the following as necessary: \( R, k, \) and \( \lambda \).

Separate system into 3 parts ②
As electric potential is a scalar, and charge density is constant in this case, part ① & ③ have the same contribution.

\[
V_1 = V_3 = \int_{-R}^{R} \frac{k \lambda dl}{l} = \int_{-R}^{R} \frac{k \lambda dl}{R} = k \lambda \ln 3
\]

For semicircle part:

\[
V_2 = \int_{-\pi}^{\pi} \frac{k \lambda R d\theta}{R} = \int_{-\pi}^{\pi} k \lambda R d\theta = k \lambda \pi
\]

Sum them up:

\[
V = V_1 + V_2 + V_3 = k \lambda (\pi + 2\ln 3)
\]