Show all work!! Report all numbers to (three) (3) significant figures.

[30 pts.] A loop of wire in the shape of a rectangle of width $w$ and length $L$ and a long, straight wire carrying a current $I$ lie on a tabletop as shown in the figure below.

(a) Determine the magnetic flux through the loop due to the current $I$. (Use any variable stated above along with the following as necessary: $\mu_0$ and $\pi$.)

(b) Suppose the current is changing with time according to $I = a + bt$, where $a$ and $b$ are constants. Determine the magnitude of the emf that is induced in the loop if $b = 2.4962$ A/s, $h = 3.25$ cm, $w = 5$ cm, and $L = 2$ m.

(c) What is the direction of the induced current in the rectangle?

(a) From definition of magnetic flux $\phi_B = \int \vec{B} \cdot d\vec{A}$

Magnetic field of a long wire: $|B| = \frac{\mu_0 I}{2\pi w}$ with direction into the page.

Area: $dA = L \, dw$.

Then flux is found as $\phi_B = \int_{w}^{w+h} \frac{\mu_0 I L}{2\pi w} \, dw = \frac{\mu_0 I L}{2\pi} \ln \left( \frac{w+h}{h} \right)$

(b) $\text{EMF} = -\frac{\partial \phi_B}{\partial t}$

Substitute $I$ by $(a+bt)$ in $\phi_B$ and take time derivative:

$\text{EMF} = -\frac{\mu_0 L a}{2\pi} \ln \left( \frac{w+h}{h} \right) = \frac{1.26 \times 10^{-6} \times 2 \times 2.4962}{2 \times 3.14} \ln \frac{8.25}{3.25} = 9.3 \times 10^{-7}$ V

(c) Due to the conservation of energy, loop generate opposite direction of magnetic field. So current is counterclockwise.
Show all work!! Report all numbers to three (3) significant figures.

[35 pts.] In the circuit diagrammed in the figure below, assume the switch has been closed for a long time interval and is opened at \( t = 0 \). Also assume \( R = 2 \, \Omega \), \( L = 2 \, \text{H} \), and \( E = 10 \, \text{V} \).

**Spts (a)** Before the switch is opened, does the inductor behave as an open circuit, a short circuit, a resistor of some particular resistance, or none of those choices? Explain.

**Spts (b)** What current does the inductor carry?

**Spts (c)** How much energy is stored in the inductor for \( t < 0 \)?

**Spts (d)** After the switch is opened, what happens to the energy previously stored in the inductor?

**Spts (e)** Sketch a graph of the current in the inductor for \( t \geq 0 \). Label the initial and final values and the time constant.

\[ +3 \]

\[ +2 \]

\[ +1 \]

\[ +1 \]

\[ +2 \]

\[ +1 \]

\[ +2 \]

\[ +2 \]

\[ +2 \]

\[ +2 \]
Show all work!! Report all numbers to three (3) significant figures.

[35 pts.] The switch in the figure below is connected to position a for a long time interval. At $t = 0$, the switch is thrown to position b. After this time, what are the following? (Let $C = 20 \, \mu F$.)

(a) the frequency of oscillation of the $LC$ circuit;
(b) the maximum charge that appears on the capacitor;
(c) the maximum current in the inductor;
(d) the total energy the circuit possesses at $t = 3.00 \, s$.

\[ \omega = \frac{1}{\sqrt{LC}} \]
\[ f = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi} \sqrt{\frac{5.055 \times 10^{-3} \times 2 \times 10^{-6}}{2 \times 10^{-6}}} \]
\[ f = 500.54 \, Hz \quad \text{Ans.} \]

(b) \[ Q_{\text{Max}} = CV_{\text{battery}} \]
\[ Q_{\text{Max}} = 2 \times 10^{-6} \times 10 \, C \]
\[ Q_{\text{Max}} = 2 \times 10^{-5} \, C \quad \text{Ans.} \]

(c) \[ E_{\text{Max}} \]
Energy conservation
\[ \frac{1}{2} L I_m^2 = \frac{1}{2} C \]
\[ I_m = Q_m \frac{1}{\sqrt{LC}} \]
\[ I_{\text{Max}} = 0.629 \, A \quad \text{Ans.} \]

(d) Total energy is conserved at any time $t$.
\[ E = \frac{1}{2} Q_m^2 = \frac{1}{2} C I_m^2 = 0.001 \, J \quad \text{Ans.} \]