Physics 2220
Spring Semester 2008
George A Williams

EXAM 4

1

Name: ELLIOTT unid: u _ _ _ _ _ _ _ _

Discussion TA (circle): Justin Mahamadou Mike Will

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Take the magnitude of the Earth's magnetic field as 1.15×10^{-4} T. If it is at an angle of 18.0° from the horizontal, calculate the magnetic flux through an area on the physics lecture table of 2.2 m^2

72° 18°

$$\Phi = BA \cdot \cos\theta = (1.15 \times 10^{-4}\ T)(2.2\ m^2)\cos(72°) = 7.82 \times 10^{-5}\ Tm^2$$

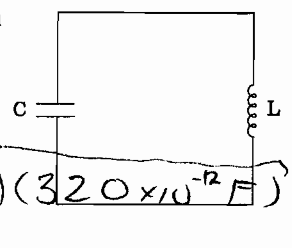
(b) For the value of the Earth's field given in (a), calculate the magnetic energy in a cube 4.25 m on a side at the earth's surface.

$$U = \frac{B^2}{2\mu_0}(Vol) = \frac{(1.15 \times 10^{-4}\ T)^2}{2(4\pi \times 10^{-7}\ Tm/A)}(4.25\ m)^3 = \underline{0.404\ J}.$$

(c) Calculate the reactance of a capacitor of 65.0 pF at a frequency (f) of 11.2 MHz.

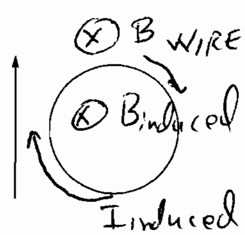
$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi(65 \times 10^{-12}\ F)(11.2 \times 10^{6}\ Hz)} = 219\ \Omega$$

(d) If the maximum current in the inductor is 17.2 mA, calculate the maximum charge on the capacitor. L = 72.0 mH, C = 320 pF.



$$\frac{1}{2}LI_m^2 = \frac{1}{2}\frac{Q_m^2}{C} \Rightarrow Q_m = I_m\sqrt{LC}$$
$$= (17.2 \times 10^{-3}\ A)\sqrt{(72 \times 10^{-3}\ H)(320 \times 10^{-12}\ F)}$$
$$= 8.26 \times 10^{-8}\ C$$

(e) A wire carries a current in the direction shown. If the magnitude of the current is decreasing, what is the direction, clockwise or counter clockwise, of the current in the loop. Both the wire and loop are in the plane of the paper.



The B field due to the wire is getting weaker so we want to reinforce it so B induced is into the page and current induced is clockwise

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Note: Only waves in vacuum travel at $3.00 \times 10^8 \frac{m}{s}$. Since $\frac{\omega}{k} = 2.45 \times 10^6 \frac{m}{s}$, this wave isn't in vacuum. This was a mistake in the problem. It would change the answers to a) and d)

The following describes the E field of an electromagnetic wave:

$$E = (4.20 \times 10^{-4}\,\text{N/C}) \cos(6.12 \times 10^2\, x + 1.50 \times 10^9\, t - \pi/3)$$

(a) Calculate the maximum value for the magnetic field for this wave.
(b) Calculate the wavelength.
(c) Calculate the frequency.
(d) Calculate the average value of the Poynting vector for the wave.
(e) Is the ware moving in the positive or negative x-direction? Explain.

The form of a traveling wave is

$$A\cos(kx - \omega t + \phi)$$

where

A = amplitude / max value

$k = \frac{2\pi}{\lambda}$ ← wavelength

$\omega = 2\pi f$ ← frequency

ϕ = phase shift

a) $E_{max} = 4.20 \times 10^{-4} \frac{N}{C}$

$$B_{max} = \frac{E_{max}}{c} = \frac{4.20 \times 10^{-4}}{3.00 \times 10^8} = 1.40 \times 10^{-12}\ T$$

b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{6.12 \times 10^2}$

$= 0.0103\ m$

c) $f = \frac{\omega}{2\pi} = \frac{1.50 \times 10^9}{2\pi}$

$= 2.39 \times 10^8\ Hz$

d) $S_{avg} = \frac{E_{max}^2}{2\mu_0 c} = \frac{(4.20 \times 10^{-4})^2}{2(4\pi \times 10^{-7})(3 \times 10^8)}$

$= 2.34 \times 10^{-10}\ W/m^2$

e) The term involving ω and k both have the same sign. Therefore, the wave is moving in the negative x-direction.

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Name: Solution

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SHOW ALL WORK!!!!!
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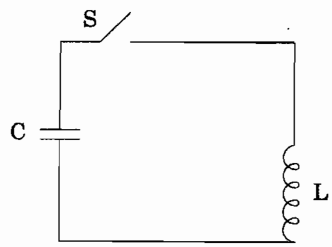
A 275 pF capacitor is charged to 175 volts. the switch is closed at t = 0. L = 950 millihenry.



5 (a) Find the frequency (f) of the oscillations in the system.
5 (b) Find the peak value of the current in the inductor.
5 (c) When the energy is equally divided between L and C, calculate the current in the inductor.
5 (d) Write an expression for the charge Q on the capacitor as a function of time. Put in **all** numerical constants.
5 (e) Write an expression for the magnitude of the current in the inductor as a function of time evaluating **all** numerical constants.

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \qquad \frac{dI}{dt} = \frac{d^2Q}{dt^2} \quad \text{so} \quad \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0 \qquad \boxed{\omega_0^2 = \frac{1}{LC}} \qquad Q(t) = Q_{max}\cos(\omega t - \phi)$$

but $\phi = 0$.

$$Q(t) = Q_{max}\cos(\omega t)$$

$$Q(0) = \mathcal{E}C = Q_{max}(1) \quad \text{thus} \quad Q_{max} = \mathcal{E}C$$

+5

(d) $$\boxed{Q(t) = \mathcal{E}C\cos(\omega t) = (175)(275\times10^{-12})\cos\left(\frac{1}{\sqrt{275\times10^{-12} * 950\times10^{-3}}}\,t\right)}$$

(e) $$\boxed{I(t) = \frac{dQ(t)}{dt} = -\mathcal{E}C\omega\sin(\omega t) = -(175)(275\times10^{-15})\left(\frac{1}{\sqrt{275\times10^{-12} * 950\times10^{-3}}}\right)\sin\left(\frac{1}{\sqrt{275\times10^{-12} * 950\times10^{-3}}}\,t\right)}$$ +5

(a) $$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = \boxed{\frac{1}{2\pi}\frac{1}{\sqrt{950\times10^{-3} * 275\times10^{-12}}}} = 9850 \text{ Hz}$$ +5

$$U_T = \frac{1}{2}\mathcal{E}^2C$$

$$\frac{1}{2}U_0 = \frac{1}{4}\mathcal{E}^2C = \frac{1}{2}LI^2$$

(b) $$I = \mathcal{E}\sqrt{\frac{C}{2L}}$$ +5

$$\boxed{I = 175\sqrt{\frac{275\times10^{-12}}{2(950\times10^{-3})}}} = .002 \text{ A}$$

$$|I_{max}| = \mathcal{E}C\omega = \boxed{(175)(275\times10^{-12})\frac{1}{\sqrt{275\times10^{-12} * 950\times10^{-3}}}}$$

$= .003$ A +5

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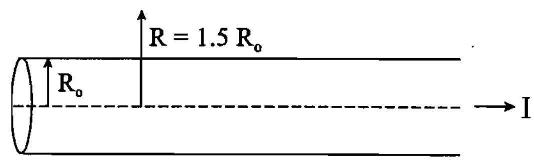
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A big, fat, long, straight wire carries a current I, with uniform current density. The wire has a radius R_o.

(a) Calculate the energy stored in the magnetic field between $R = R_o$ and $R = 1.5\ R_o$, for a length ℓ of the wire.

(b) Calculate the energy stored in the magnetic field inside the wire for a length ℓ of the wire.



Solution:

a) Energy:

$$dE = \frac{B^2}{2\mu_0} dv \Rightarrow E = \int_v \frac{B^2}{2\mu_0} dv \quad \text{where } B = \frac{\mu_0 I}{2\pi r}$$

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$$E = \frac{1}{2\mu_0}\int \left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi r\, dr\, \ell = \frac{\mu_0 I^2}{4\pi}\ell \int_{R_0}^{1.5R_0} \frac{dr}{r} = \boxed{\frac{\mu_0 I^2}{4\pi}\ell \ln(1.5)}$$

b) Similary:

$$dE = \frac{B^2}{2\mu_0} dv$$

Inside the wire: $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc} = \mu_0 \int J\, da$

$$B \cdot 2\cancel{\pi} r = \mu_0 J \cancel{\pi} r^2 \Rightarrow B = \frac{\mu_0 J}{2} r$$

or $B = \dfrac{\mu_0 I}{2\pi R_0^2} r$ 9

Hence

$$E = \frac{1}{2\mu_0}\int \left(\frac{\mu_0 I\, r}{2\pi R_0^2}\right)^2 2\pi r\, dr\, \ell$$

$$= \frac{\mu_0 I^2 \ell}{4\pi R_0^4}\int_0^{R_0} r^3 dr = \frac{\mu_0 I^2}{4\pi R_0^4}\ell \frac{R_0^4}{4}$$

$$\boxed{E = \frac{\mu_0 I^2 \ell}{16\pi}}$$

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