Physics 172
Winter Quarter 1986
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FIRST MIDTERM

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REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

(a) Calculate magnitude of the electric field 12.5 m from a point charge of value $+18.7 \times 10^{-3}$ C.

$E = 1.077 \cdot 10^{6}$ N/C

(b) Calculate the electric force between a proton and an electron that are 0.50×10^{-10} m apart. (The proton charge is equal in magnitude to the the electron charge, but opposite in sign.)

$F = -9.22 \cdot 10^{-8}$ N attractive

(c) Calculate the first three terms of binomial expansion for the expression below, for $a \ll x$.

$$\frac{1}{\left(x^2 + a^2\right)^{-7/2}} \approx \frac{1}{x^7} - \frac{7}{2}\frac{a^2}{x^9} + \frac{63}{8}\frac{a^4}{x^{11}}$$

(d) Calculate $|g|$ at a point 1/4 the distance from the center of the earth to the surface. (Assume uniform density.)

$|g| = 2.45 \frac{m}{sec^2}$

(e) Calculate the gravitational force on a 1.00 kg object that is two earth radii above the earth's surface.

$F = 1.089$ N attractive

a) The electric field of a point charge is given by $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$. Set $\frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \frac{Nm^2}{C^2}$ (1)
and use the values given for r and q. $\overset{(1)}{\Rightarrow}$

$\boxed{E = 1.077 \cdot 10^{6} N/C}$

b) The electrostatic force between two pointcharges has a magnitude of $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 \cdot q_2}{r^2}$. Using the charge $q_1 = 1.6 \cdot 10^{-19} C$ for a proton and $q_2 = -1.6 \cdot 10^{-19} C$ for an electron, gives us

$\boxed{F = -9.22 \ 10^{-8} N}$ (minus sign indicates attractive force!)

c) $\frac{1}{(x^2+a^2)^{7/2}} = \frac{1}{x^7(1+\frac{a^2}{x^2})^{7/2}} = \frac{1}{x^7(1+y)^{7/2}}$ where $y = \frac{a^2}{x^2}$ (2)

Consider a function $f(y)$. The Taylor-series expansion of this function is given as: $f(y) = f(0) + f'(0) \cdot \frac{y}{1!} + f''(0) \frac{y^2}{2!} + \ldots$
In general $f(y) = \sum_{n=0}^{\infty} f^n(0) \frac{y^n}{n!}$ where f^n is the n'th derivative with respect to y! (3)

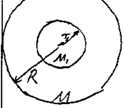
Set $f(y) = \frac{1}{(1+y)^{7/2}} = (1+y)^{-7/2}$ it follows from eq. (3)

$f(y) \approx 1 - \frac{7}{2} y + \frac{63}{8} \cdot y^2$. Combining eq. (2) and (4) (4)

gives us $\frac{1}{(x^2+a^2)^{7/2}} \approx \frac{1}{x^7} \cdot (1 - \frac{7}{2} y + \frac{63}{8} y^2) = \frac{1}{x^7}(1 - \frac{7}{2}\frac{a^2}{x^2} + \frac{63}{8}\frac{a^4}{x^4})$

So $\boxed{\frac{1}{(x^2+a^2)^{7/2}} \approx \frac{1}{x^7} - \frac{7}{2}\frac{a^2}{x^9} + \frac{63}{8}\frac{a^4}{x^{11}}}$

d) Sketch:



Let the earth have a total mass M and a radius R. The mass density, since uniform, is given as

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}. \qquad (1)$$

Now due to the spherical symmetry the gravitational field at r_1 is given by

$$g = G \cdot \frac{M_1}{r_1^2}. \qquad (2)$$

Where M_1 is the mass contained in the sphere of radius r_1. Since uniform density

$$\rho = \frac{M_1}{V_1} = \frac{M_1}{\frac{4}{3}\pi r_1^3} \text{ or from eq. (1): } M_1 = M \cdot \frac{r_1^3}{R^3} \qquad (3)$$

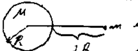
inserting into eq. (2) gives us: $g = G \cdot M \frac{r_1}{R^3}$ (4)

Now $r_1 = \frac{1}{4} R$, so $\overset{(4)}{\Rightarrow}$ $g = \frac{1}{4} \cdot GM/R^2$. (5)

It is easy to show that $g_s = \frac{GM}{R^2}$ the gravitational field at the earth surface! so from eq. (5)

$$\boxed{g = \frac{g_s}{4} = \frac{9.80}{4} \frac{m}{sec^2} = 2.45 \frac{m}{sec^2}}$$

c) Sketch:



Consider the earth of spherical symmetric mass distribution. We then can consider it as a mass point at the center of mass M. Then Newton's law of gravitation gives us for the magnitude $F = G \cdot \frac{M \cdot m}{r^2}$ but $r = 3R$, so that $F = G \cdot \frac{Mm}{9R^2}$. Now $g_s = G\frac{M}{R^2}$, so

$F = g_s \cdot m/9$ but $g_s = 9.8 \frac{m}{sec^2}$, so

$$\boxed{F = \frac{1kg \cdot 9.8 \frac{m}{sec^2}}{9} = 1.089\ N \text{ attractive}}$$