Physics 172
Winter Quarter 1986
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George Williams

310

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FIRST MIDTERM

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Discussion Instructor (Circle One): Brown Chung Pollard Rothman

Discussion Section #: _____ Schweizer Soderberg Vaseghi Viohl

REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

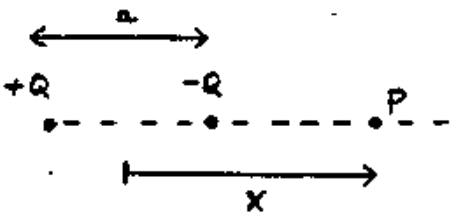
Two charges of equal magnitude, +Q and -Q are a distance a apart on the x-axis. Take x = 0 at the midpoint between the two charges.

(a) Write a general expression for the electric field at point P, at the coordinate x.
(b) Using the binomial expansion, calculate the electric field at P, keeping the first two non-zero terms involving a.
(c) If a/x is 0.10, what fractional error is introduced by ignoring the last term in part (b). (That is, what fraction of the total answer is this last term?)

(7)

a) $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\vec{E} = \left[\frac{1}{4\pi\epsilon_0}\frac{Q}{(x+\frac{a}{2})^2} - \frac{1}{4\pi\epsilon_0}\frac{Q}{(x-\frac{a}{2})^2}\right]\hat{x}$$



$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{(x+\frac{a}{2})^2} - \frac{1}{(x-\frac{a}{2})^2}\right]\hat{x}}$$

b) $$E = \frac{Q}{4\pi\epsilon_0}\frac{1}{x^2}\left[\frac{1}{(1+\frac{a}{2x})^2} - \frac{1}{(1-\frac{a}{2x})^2}\right]$$

(13)

$$(1+\tfrac{a}{2x})^{-2} = 1 + (-2)(\tfrac{a}{2x}) + \frac{(-2)(-3)}{2!}(\tfrac{a}{2x})^2 + \frac{(-2)(-3)(-4)}{3!}(\tfrac{a}{2x})^3 + \cdots$$

$$= 1 - \frac{a}{x} + \frac{3}{4}\frac{a^2}{x^2} - \frac{1}{2}\frac{a^3}{x^3} + \cdots$$

$$(1-\tfrac{a}{2x})^{-2} = 1 + (-2)(-\tfrac{a}{2x}) + \frac{(-2)(-3)}{2!}(-\tfrac{a}{2x})^2 + \frac{(-2)(-3)(-4)}{3!}(-\tfrac{a}{2x})^3 + \cdots$$

$$= 1 + \frac{a}{x} + \frac{3}{4}\frac{a^2}{x^2} + \frac{1}{2}\frac{a^3}{x^3} + \cdots$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x^2}\left(-\frac{2a}{x} - \frac{a^3}{x^3}\right)\hat{x} \Rightarrow \boxed{\vec{E} = -\frac{Q}{4\pi\epsilon_0}\frac{1}{x^2}\left(\frac{2a}{x} + \frac{a^3}{x^3}\right)\hat{x}}$$

c) $\frac{a}{x} = .1$

$$\text{fractional error} = \frac{a^3/x^3}{2a/x + a^3/x^3} = \frac{(.1)^3}{2(.1)+(.1)^3} = \frac{.001}{.201}$$

(5)

$$\boxed{\text{Fractional error} = .005}$$