Winter Quarter 1989
January 20, 1989
George Williams

4

FIRST EXAM

Name (print) Doug Ball Name (signed) X

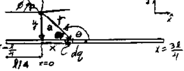
Discussion Instructor (circle one): Cady McAllister Molina Stone

Discussion Section #: Ø

AVERAGE = ~~8.33~~ 11.
Standard deviation = 8.33

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given a line of charge with charge density $\lambda = +3.75\ \mu C/m$ (3.75×10^{-6} C/m), and length $\ell = 1.27$ m. Calculate the electric field (magnitude and direction) at point P located at distance a from the line, and distance $\ell/4$ from one end of the line. Take a = 0.180 m, and calculate a numerical value complete with units.



$\vec{E} = K\int \frac{dq}{r^2}\hat{r}$ and $dq = \lambda dx$ also $r^2 = a^2 + x^2$ and $\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y}$

Note: $\cos\theta = \frac{-x}{r}$ because θ is measured from $+\hat{x}$ not $-\hat{x}$.
and $\sin\theta = \frac{a}{r}$.

$\vec{E}_x$ is : $\vec{E}_x = K\int_{-\frac{\ell}{4}}^{\frac{3\ell}{4}} \frac{\cos\theta\,\lambda dx}{r^2}\hat{x}$. substituting in $\cos\theta = \frac{-x}{r}$ and $r^2 = a^2 + x^2$,

$\vec{E}_x = -K\lambda\int_{-\frac{\ell}{4}}^{\frac{3\ell}{4}} \frac{x\,dx}{(a^2+x^2)^{3/2}}\hat{x}$. From the integral table (eq. 123),

$$\boxed{\vec{E}_x = -K\lambda\left[\frac{-1}{\sqrt{(\frac{3\ell}{4})^2+a^2}} + \frac{1}{\sqrt{(\frac{1}{4}\ell)^2+a^2}}\right]\hat{x}} \quad \boxed{|\vec{E}_x| = -5.77\times10^{4}\,N/C}$$

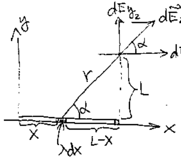
$\vec{E}_y$ is : $\vec{E}_y = K\lambda\int_{-\frac{\ell}{4}}^{\frac{3\ell}{4}} \frac{\sin\theta\,dx}{r^2}\hat{y} \Rightarrow \vec{E}_y = K\lambda a\int_{-\frac{\ell}{4}}^{\frac{3\ell}{4}} \frac{dx}{(a^2+x^2)^{3/2}}\hat{y}$. From the

table (eq. 122), $$\boxed{\vec{E}_y = K\lambda\left[\frac{\frac{3\ell}{4}}{a\sqrt{(\frac{3\ell}{4})^2+a^2}} + \frac{\frac{\ell}{4}}{a\sqrt{(\frac{\ell}{4})^2+a^2}}\right]\hat{y}} \quad \boxed{|\vec{E}_y| = 3.47\times10^{5}\,N/C}$$

$|\vec{E}| = \sqrt{|\vec{E}_x|^2 + |\vec{E}_y|^2}$
$\boxed{|\vec{E}| = 3.52\times10^{5}\,N/C}$

$\phi = \tan^{-1}\frac{|\vec{E}_y|}{|\vec{E}_x|} \Rightarrow \boxed{\phi = 80.6^\circ \text{ from } -\hat{x} \text{ axis}}$

Problem 4 P2



$$\cos\alpha = \frac{L-x}{r} = \frac{L-x}{\{(L-x)^2+L^2\}^{1/2}}$$

$$\sin\alpha = \frac{L}{r} = \frac{L}{\{(L-x)^2+L^2\}^{1/2}}$$

$$\vec{E}_{x_2} = \int dE_{x_2} = k\int \frac{\lambda\, dx}{r^2}\cos\alpha = k\int_0^L \frac{\lambda(L-x)\,dx}{\{(L-x)^2+L^2\}^{3/2}}\ \vec{i} \qquad +4$$

$$\vec{E}_{y_2} = \int dE_{y_2} = k\int \frac{\lambda dx}{r^2}\sin\alpha = k\int_0^L \frac{\lambda L\, dx}{\{(L-x)^2+L^2\}^{3/2}}\ \vec{j} \qquad +4$$

$$\vec{E} = (\vec{E}_{x_1}+\vec{E}_{x_2}) + (\vec{E}_{y_1}+\vec{E}_{y_2}) = k\lambda\int_0^L \frac{(L+L-x)}{\{(L-x)^2+L^2\}^{3/2}}\,dx\ \vec{i}\ +$$

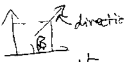
$$+k\lambda\int_0^L \frac{(L-x+L)\,dx}{\{(L-x)^2+L^2\}^{3/2}}\ \vec{j}$$

$$\vec{E} = |\vec{E}_x|^2+|E_y|^2 = k\lambda\int_0^L \frac{(L+L-x)}{\{(L-x)^2+L^2\}^{3/2}}\,dx\,(1^2+1^2)^{1/2} = \sqrt{2}k\lambda\int_0^L \frac{(2L-x)\,dx}{\{(L-x)^2+L^2\}^{3/2}}$$

+4

Direction  use symmetric property

or $\tan\beta = \frac{|E_y|}{|E_x|} = 1 \Rightarrow \beta = 45°$



+5