Physics 2220
Spring 2001
George A. Williams

FIRST MIDTERM

3

Name: Jiansheng Wu

Discussion Instructor (circle): Billeter Diwekar Kennedy Murray Whitaker

Discussion Section # ____________ Student ID #: ____________

SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A spherically symmetric charge distribution on a non-conductor is modeled by a charge density given by:

$$\rho(r) = \frac{B}{r},$$

where B is a constant. The charge distribution has an outer radius R_o. There is no charge outside of R_o.

(a) Calculate the total charge Q.
(b) Calculate the electric field at the interior point $r = R_o/3$.

(a)
$$Q = \int \rho(r)\,dV$$
$$= \int_0^{R_o} \rho(r)\,4\pi r^2\,dr$$
$$= \int_0^{R_o} \frac{B}{\cancel{r}}\,4\pi r^{\cancel{2}}\,dr$$
$$= 4\pi B \int_0^{R_o} r\,dr$$
$$= 4\pi B \left.\frac{r^2}{2}\right|_0^{R_o}$$
$$= \boxed{2\pi B R_o^2}$$

(b)
$$\int \vec{E}\cdot d\vec{s} = \frac{Q_{enclosed}\left(\frac{R_o}{3}\right)}{\varepsilon_o}$$
$$\int \vec{E}\cdot d\vec{s} = 4\pi r^2 E \qquad r = \frac{R_o}{3}$$
$$Q_{enclosed}\left(\frac{R_o}{3}\right) = 2\pi B\left(\frac{R_o}{3}\right)^2$$
$$\therefore\quad E = \frac{2\pi B\left(\frac{R_o}{3}\right)^2}{4\pi\left(\frac{R_o}{3}\right)^2\varepsilon_o} = \boxed{\frac{B}{2\varepsilon_o}}$$

or

$$E = \frac{kQ_{enclose}}{r^2} \quad \left(\frac{kQ_{enclose}\,\vec{r}}{r^3}\right)$$
$$= \frac{k\left[2\pi B\left(\frac{R_o}{3}\right)^2\right]}{\left(\frac{R_o}{3}\right)^2} = \boxed{2\pi k B}$$

this can be done only because of the charge is spherically symmetric