393; Average 16.5; 6.6

4

Physics 172
Winter Quarter 1983
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George Williams

SECOND MIDTERM

Name: _____

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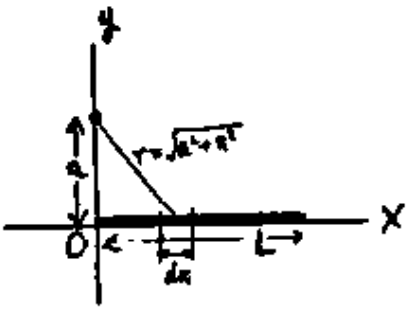
Discussion Section # _____

PROBLEM 4

On a thin rod of length L lying along the x-axis with one end at the origin (x = 0), as shown in the figure, there is distributed a charge per unit length given by $\lambda = kx$, where k is a constant.

(a) Taking the electrostatic potential at infinity to be zero, find V at the point P on the y-axis. P is a distance a from the origin.
(b) Determine the vertical component, E_y, of the electric field intensity at P from the result of part (a).



[Solution]

(a). $$V = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\,dx}{\sqrt{a^2+x^2}} = \frac{k}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{\sqrt{x^2+a^2}}$$

$$= \frac{k}{4\pi\varepsilon_0}\sqrt{x^2+a^2}\,\Big|_0^L = \frac{k}{4\pi\varepsilon_0}\left(\sqrt{L^2+a^2} - |a|\right).$$

Here $a > 0$. So we have

$$\boxed{V = \frac{k}{4\pi\varepsilon_0}\left(\sqrt{L^2+a^2} - a\right) \qquad (\text{for } a>0)}$$

(b). $$E_y\Big|_P = -\frac{\partial V(a)}{\partial y}\Big|_{y=a} = -\frac{\partial V(a)}{\partial a} = \frac{k}{4\pi\varepsilon_0}\left(1 - \frac{a}{\sqrt{L^2+a^2}}\right).$$

$$\boxed{E_y\Big|_P = \frac{k}{4\pi\varepsilon_0}\left(1 - \frac{a}{\sqrt{L^2+a^2}}\right).}$$