Given a long cylinder of a non-conducting material. This cylinder is charged with \( \lambda = 2.75 \times 10^{-8} \text{ C/m} \). The volume charge density is given by

\[ \rho = \rho_0 \left(1 - \alpha R^{3/2}\right) \]

where the charge density goes to zero at \( R = R_0 = 3.00 \text{ cm} \), the outside surface of the rod. If \( \alpha \) and \( \rho_0 \) are constants, find \( \lambda \). 

(a) Numerical values for \( \alpha \) and \( \rho_0 \).

(b) The electric field at \( R = 3/4 R_0 \).

(c) The electric potential difference between the center and outside surface of the rod.

\[ \lambda = \frac{1}{k^{3/2}} = 192.5 \text{ m}^{-3/2} \]

\( a) \quad \rho(R) = C \Rightarrow \lambda = \frac{1}{k^{3/2}} = 192.5 \text{ m}^{-3/2} \)

\[ 2\lambda = \int_0^{2\pi} d(\text{volume}) \]

\[ d(\text{volume}) = \lambda 2\pi R \, dR \]

\[ \lambda = \int_0^{2\pi} \rho 2\pi R \, kR \, dR \]

\[ \lambda = \int_0^{2\pi} 2\pi \rho (1 - k^{3/2}) \, kR \, dR = 2\pi \rho R^2 \left[\frac{k}{k^{3/2}} - \frac{3}{2}\right] \text{ using } k = \frac{1}{k^{3/2}} \]

\[ \rho_0 = \frac{2.75 \times 10^{-8}}{2\pi R_0^2} = 2.27 \times 10^{-5} \text{ C/m}^2 \]

\[ k = 193 \text{ m}^{-3/2} \]

5 points each
b) Gauss' Law:  \[ \oint \vec{E} \cdot d\vec{a} = \frac{Q_{\text{net}}}{\epsilon_0} \]

Gaussian Surface - cylinder of radius \( r \), concentric with charged cylinder, of length \( L \):

\[ \oint \vec{E} \cdot d\vec{a} = \int_{-L/2}^{L/2} \vec{E} \cdot d\vec{l} + \int_{-L/2}^{L/2} \vec{E} \cdot d\vec{l} + \int_{-L/2}^{L/2} \vec{E} \cdot d\vec{l} = E(2\pi L) \cos \theta + \int_{-L/2}^{L/2} \vec{E} \cdot d\vec{l} \]

\[ E(2\pi L) = \frac{Q_{\text{net}}}{\epsilon_0} \]

\[ \varepsilon_{\text{net}} = \int_0^L \rho d\ell = \int_0^L \rho_0 \left( 1 - \frac{r^2}{R^2} \right) L \ 2\pi R \ \ell R \]

\[ E(2\pi L) = 2\pi \rho_0 L \left[ \frac{\ell^2}{2} - \frac{\ell^3}{3} \right] \frac{1}{\ell} \]

\[ \Rightarrow E(r) = \frac{\rho_0}{\epsilon_0} \left[ \frac{\ell^2}{2} - \frac{\ell^3}{3} \right] \]

\[ E\left( \frac{L}{2} \right) = 1.81 \times 10^6 \text{ V/m} \]

c) \[ dV = \int_0^L \int_0^{2\pi} \int_0^r \vec{E} \cdot d\vec{a} \cos \theta \ d\theta \ d\phi \ dr \]

\[ dV = \frac{\rho_0}{\epsilon_0} \int_0^L \int_0^{2\pi} \int_0^r \left[ \frac{\ell^2}{2} - \frac{\ell^3}{3} \right] \ d\theta \ d\phi \ dr = \frac{\rho_0}{\epsilon_0} \ell^5 \left( \frac{1}{4} - \frac{1}{11} \right) \]

\[ dV = 3.89 \times 10^{-2} \text{ V} \]

Major Problems:

1) Time - the charge will account for this
2) \( \rho = 0 \) - this makes no sense \( \checkmark \)
3) \( \alpha \) given numerically as \( \frac{T}{m} \) -1 pt
4) units; significant figures -2 pt/each time
5) Volume integrals - draw a picture of your differential volume, this makes it easier to see. Some people had their d(volume) with dimensions of length or area - this is wrong.
6) Rolling formulas off instead that did not apply