Physics 172
Winter Quarter 1987
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George Williams

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SECOND MIDTERM

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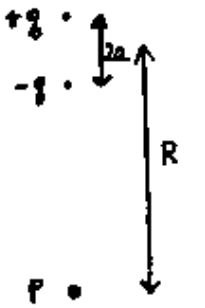
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given an electric dipole consisting of a charge q = +3.75 μC a distance 2a from a charge of q = -3.75 μC. Take a = 1.00 mm. Use the binomial theorem to calculate the electric potential at a distance R of 3.75 m from the center of the dipole along the axis of the dipole.



Solution:

Step 1 (+5)
$$U = \frac{kq}{R+a} + \frac{k(-q)}{R-a}$$

Step 2 (+15)
$$= kq\left[\frac{R-a-R-a}{R^2-a^2}\right] = \frac{-2akq}{R^2}\left[1-\left(\frac{a}{R}\right)^2\right]^{-1}$$
$$= -\frac{2akq}{R^2}\left[1 + \frac{(-1)\left(-\frac{a^2}{R^2}\right)}{1!} + \frac{(-1)(-2)\left(-\frac{a^2}{R^2}\right)^2}{2!} + \cdots\right]$$
$$= -\frac{2akq}{R^2}\left[1 + \frac{a^2}{R^2} + \frac{a^4}{R^4} + \cdots\right]$$

Step 3 (+5)
$$\therefore \doteq -\frac{2akq}{R^2} = -\frac{2(9\times10^9)(3.75\times10^{-6})\times(0.001)}{(3.75)} \qquad \boxed{\because R \gg a}$$
$$= -4.80\ (V)$$

the binomial theorem

$$(1+x)^n = \sum_{r=0}^{n} \frac{n!}{(n-r)!\,r!}x^r = 1 + \frac{n}{1!}x + \frac{n(n-1)}{2!}x^2 + \cdots$$

or:
$$U = \frac{kq}{R+a} + \frac{k(-q)}{R-a}$$
$$= \frac{kq}{R}\left[\left(1+\frac{a}{R}\right)^{-1} - \left(1-\frac{a}{R}\right)^{-1}\right]$$
$$= \frac{kq}{R}\left[\left(1 - \frac{a}{R} + \frac{(-1)(-2)}{2}\left(\frac{a}{R}\right)^2 + \frac{(-1)(-2)(-3)}{2\cdot3}\left(\frac{a}{R}\right)^3 + \cdots\right)\right.$$
$$\left. - \left(1 + \frac{a}{R} + \frac{(-1)(-2)}{2}\left(-\frac{a}{R}\right)^2 + \frac{(-1)(-2)(-3)}{2\cdot3}\left(-\frac{a}{R}\right)^3 + \cdots\right)\right]$$
$$= -\frac{2akq}{R^2}\left[1 + \frac{a^2}{R^2} + \cdots\right]$$
$$= -4.80\ (V)$$

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Grading explaination :

In Problem 2, if one had not used the binomial theorem, one lost 15 points, even though one can get the correct numerical answer.

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example :

+5 $\left\{ \begin{array}{l} U = \dfrac{kq}{R+a} - \dfrac{kq}{R-a} \\ \quad = kq\left[\dfrac{1}{R+a} - \dfrac{1}{R-a}\right] \end{array} \right.$

+5 $\left\{ \begin{array}{l} = 9\times10^{9} \times 3.75 \times 10^{-6} \left[\dfrac{1}{3.751} - \dfrac{1}{3.749}\right] \\ = -4.8\ (V) \end{array} \right.$