N = 166

SECOND MIDTERM

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Discussion Instructor (circle one): Baselgia Morrill Reeve Stoops Zhang

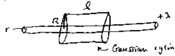
Discussion Section # ______

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

Given a very long, thin wire of radius r and a charge density of $+\lambda$ c/m.

(a) Calculate the electric potential difference between the surface of the wire, and a point at a distance R away from the center of the wire where $R > r$.
(b) State clearly the sign of the potential difference, $V(R) - V(r)$, and give a physical reason for it.

20 pts a) First calculate the electric field a distance $R > r$ from the center of the wire.



Since the electric field is purely radial for a sufficiently long wire, it is constant on the surface of the cylinder, and hence may be removed from the flux integral.

$$\Rightarrow E \cdot 2\pi R \ell = \frac{\lambda \ell}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi R \epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{+\lambda}{2\pi R\epsilon_0}\hat{r}}$$ 10 pts

Now integrate the field to get the electric potential:

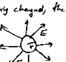
$$V = -\int \vec{E}\cdot d\vec{s} \quad \text{here } d\vec{s} = dr\,\hat{r} \quad \therefore V(R) - V(r) = -\int_r^R \frac{\lambda}{2\pi r' \epsilon_0}dr' = \boxed{\frac{-\lambda}{2\pi\epsilon_0}\ln\frac{R}{r} = V(R) - V(r)}$$

10 pt

5 pts b) Since $R > r$, $\ln\frac{R}{r} > 0$ $\therefore$ $\boxed{V(R) - V(r) \text{ is negative}}$

Why? Since the wire is positively charged, the electric field is radially outward:

e.g. in cross-section



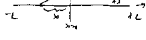
Hence in moving a positive test charge from R to r one would have to do positive work against the field.

Since work is equal to the change in potential energy, $W = U_r - U_R > 0$

$\Rightarrow V(r) > V(R) \Rightarrow V(R) - V(r) < 0$.

One can also think of it in terms of just potentials: as one approaches a positive charge, the potential due to that charge increases ($V = \frac{kq}{r}$ for a point charge). $\therefore V(r) > V(R)$.

NOTE: Several people (all unsuccessfully) attempted part (a) by a direct calculation of the potential. This is the way to do it: Let the wire be of length 2L with $L \gg R, r$.



$V(r) = \int_{-L}^{L}\frac{k\lambda dx}{\sqrt{r^2+x^2}} = V(r) = 2k\int_0^L \frac{\lambda\, dx}{(r^2+x^2)^{1/2}} = 2k\lambda \ln\left(x+\sqrt{x^2+r^2}\right)\Big|_0^L = 2k\lambda\left[\ln\left(L+\sqrt{L^2+r^2}\right) - \ln r\right]$

both sides

Do the same for R. These gives the potentials relative to 0 at ∞.

Then $V(R) - V(r) = 2k\lambda\left[\ln\frac{L+\sqrt{L^2+R^2}}{L+\sqrt{L^2+r^2}} - \ln\frac{R}{r}\right] \approx -2k\lambda\ln\frac{R}{r}$ as before

≈ ln 1 = 0 for L ≫ R, r