SECOND MIDTERM

SHOW ALL WORK!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A sphere of nonconductor has a charge distribution given by

$$\rho = \rho_0 (1 - \alpha R^3)$$

for $R < R_0$, where $R_0$ is the radius of the sphere. $\rho = 0$ for $R > R_0$. The total charge on the sphere is $Q$. The value of $\rho$ goes to zero at $R = R_0$.

(a) Calculate the electric field at any point within the sphere, a distance $R$ from its center.

(b) Assume that $Q$ and $R_0$ are known. Find a formula for $\alpha$.

(c) Assume that $Q$ and $R_0$ are known. Find a formula for $\rho_0$.

(d) Calculate the magnitude of electric potential difference between $R_0$ and $R$ (inside the sphere) assuming the charge is positive.

(a) Using Gauss's Law

$$E \cdot A = \frac{\rho_{\text{cn}}}{\varepsilon_0}$$

$$4\pi R^2 E = \frac{1}{\varepsilon_0} \int_0^R \rho_0 (1 - \alpha R^3) 4\pi R^2 dR$$

$$= \frac{4\pi \rho_0}{\varepsilon_0} \left[ \frac{R^3}{3} - \frac{\alpha R^6}{6} \right]_0^R$$

that is

$$E = \frac{\frac{\rho_0 R}{3\varepsilon_0}}{1 - \frac{\alpha R^3}{2}}$$

radially outward

(b) Since the value of $\rho$ goes to zero at $R = R_0$.

therefore

$$\rho_0 (1 - \alpha R_0^3) = 0 \quad \Rightarrow \quad \alpha = \frac{1}{R_0^3}$$

(c)

$$Q = \int_0^{R_0} \rho dV = 4\pi \int_0^{R_0} \rho_0 (1 - \alpha R^3) R^2 dR$$

$$= \frac{4\pi \rho_0}{3} (R_0^3 - \frac{\alpha}{2} R_0^6)$$

Plug in $\alpha = \frac{1}{R_0^3}$, can get

$$\rho_0 = \frac{3Q}{2\pi R_0^3}$$

(d) \[ 10 \text{ points} \]

$$\Delta V = V(R) - V(R_0) = -\int_{R_0}^{R} \vec{E} \cdot d\vec{S}$$

$$\Delta V = -\frac{\rho_{\text{cn}}}{3\varepsilon_0} \int_{R_0}^{R} (R - \frac{\alpha}{2} R^4) dR = -\frac{\rho_{\text{cn}}}{3\varepsilon_0} \left[ \frac{R^2}{2} - \frac{\alpha}{10} R^5 \right]_{R_0}^{R}$$

$$= \frac{Q}{2\varepsilon_0} \left[ \frac{R^3}{3} - \frac{\alpha R^6}{6} \right]$$

$$= \frac{Q}{2\varepsilon_0} \left[ R_0^3 - R^3 - \frac{1}{3} \frac{R_0^3}{R^3} (R_0^3 - R^3) \right]$$