

# FIRST MIDTERM

3

Name: \_\_\_\_\_

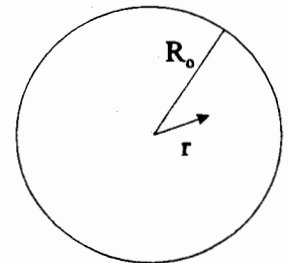
Discussion Instructor (circle): Billeter      Blake      Gillman      Herring

Student ID #: \_\_\_\_\_

**SHOW ALL WORK!!!!**  
**REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!**  
**Use the conversion constants and data given on the front page.**

A disk of radius  $R_0$  is charged with a surface charge density given by  $\sigma = \sigma_0 r$ . The units of  $\sigma$  are  $C/m^2$ .

- (a) 10' Calculate the total charge on the disk.  
 (b) 15' Calculate the electric field on the axis of the disk a distance  $x$  away from the disk (out of the paper a distance  $x$ ).



$$\begin{aligned}
 (a) \quad Q &= \int \sigma da && 1' \\
 &= \int_0^{R_0} \sigma_0 r \cdot 2\pi r dr && 7' \\
 &= 2\pi\sigma_0 \times \frac{r^3}{3} \Big|_0^{R_0} \\
 &= \frac{2\pi}{3} \sigma_0 R_0^3 && 2'
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad dq &= 2\pi r \sigma_0 r dr && 4' \\
 \vec{E} &= \int_0^{R_0} \frac{k dq}{r^2} \hat{r} \\
 &= \int_0^{R_0} \frac{k \cdot 2\pi r \sigma_0 r dr}{(r^2 + x^2)} \times \frac{x}{\sqrt{r^2 + x^2}} \cdot \hat{x} && (6' \text{ for } \vec{r}, 4' \text{ for } r^2) \\
 &= \int_0^{R_0} \frac{2\pi k x \sigma_0 r^2 dr}{(r^2 + x^2)^{3/2}} \hat{x} \\
 &= 2\pi k x \sigma_0 \left[ \frac{-r}{\sqrt{r^2 + x^2}} + \ln(r + \sqrt{r^2 + x^2}) \right]_{r=0}^{r=R_0} \hat{x} \\
 &= 2\pi k x \sigma_0 \left[ \frac{-R_0}{\sqrt{R_0^2 + x^2}} + \ln(R_0 + \sqrt{R_0^2 + x^2}) - \ln x \right] \hat{x} && 1'
 \end{aligned}$$

$\hat{x}$  is pointing outward.