George A. Williams

SECOND MIDTERM

4

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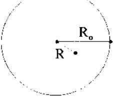
SHOW ALL WORK!!!!!
REPORT ALL NUMBERS TO THREE SIGNIFICANT FIGURES!
Use the conversion constants and data given on the front page.

A spherical distribution of positive charge has a charge density given by $\rho(R) = \rho_o R^3$ for $R < R_o$ where ρ_o is a constant.



(a) Calculate the electric field at an arbitrary interior point of radius R.
(b) Find the magnitude of the potential difference between the center and the surface of the sphere.
(c) If the total charge on the sphere is 1.60×10^{-6} C, and the radius of the sphere is 0.875 m, find the potential difference in (b) in volts.

a) $\oint \vec{E} d\vec{A} = \frac{q_{in}}{\varepsilon_o}$

$$E(R) 4\pi R^2 = \frac{1}{\varepsilon_o}\int_0^R \rho(r) 4\pi r^2 dr = \frac{4\pi\rho_o}{\varepsilon_o}\int_0^R r^5 dr = \frac{4\pi\rho_o}{\varepsilon_o}\cdot\frac{R^6}{6}$$

$$E(R) = \frac{\rho_o R^4}{6\varepsilon_o}$$

b) $$|V| = \left|-\int \vec{E} d\vec{l}\right| = \int_0^{R_o} E(R) dR = \frac{\rho_o}{6\varepsilon_o}\int_0^{R_o} R^4 dR = \frac{\rho_o R^5}{30\varepsilon_o}$$

c) $$Q = \int \rho dV = \int_0^{R_o} \rho_o R^3 \cdot 4\pi R^2 dR = \frac{4\pi\rho_o R_o^6}{6} \Rightarrow \rho_o = \frac{3Q}{2\pi R_o^6}$$

$$V = \frac{Q}{20\pi\varepsilon_o R_o} = \frac{kQ}{5R_o} = 3.29\cdot 10^3 \text{ V} = 3.29 \text{ kV}$$