Consider a sphere of non-conductor, whose radius is $R_0$ and $\kappa = 1.00$. It is positively charged, with a charge density given by $\rho = A/R$, where $A$ is a constant, and $R$ the distance from the center of the sphere. Calculate the energy stored in the electric field between $R = R_0/2$ and $R = R_0$.

Use Gauss's Law to find $E$-field

$$\int E \cdot dA = \frac{Q}{\varepsilon_0} = \int \rho dV_0 \Rightarrow \int E \cdot dA = \frac{1}{\varepsilon_0} \int \rho dV_0$$

$$E(R) = \frac{A}{2\varepsilon_0} R \quad \Rightarrow \quad E = \frac{A}{2\varepsilon_0} \text{ for } R < R_0$$

$$U_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \varepsilon_0 \frac{A^2}{4\varepsilon_0} = \frac{A^2}{8\varepsilon_0} \quad \kappa = 1.00$$

$$U = \int U_E dV_0 = \frac{A^2}{8\varepsilon_0} \int_{R_0/2}^{R_0} 4\pi R^2 dR = \frac{A^2\pi}{6\varepsilon_0} \left[ R^3 \right]_{R_0/2}^{R_0}$$

$$U = \frac{A^2\pi}{6\varepsilon_0} \left[ R_0^3 - \frac{R_0^3}{8} \right] = \frac{A^2\pi}{6\varepsilon_0} \left[ \frac{7}{8} R_0^3 \right]$$

$$U = \frac{7A^2\pi}{48\varepsilon_0} R_0^3$$

Dimensions: $[U] = \left( \frac{N \cdot m^2}{C^2} \right) \left( \frac{C^2}{m^2} \right) (m^3) = N \cdot m$